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Relaxation Oscillations

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I. MECHANICAL

1. Probably the most readily visualized example of a mechanical system capable of performing relaxation oscillations is afforded by the siphoning cistern, or, for those who refuse to regard any apparatus as scientific unless encountered in a laboratory, the Soxhlet extractor of the chemists.

Water is fed into a cistern, the cross-sectional area S of which we may assume to be constant, at a uniform rate Q determined by the setting of the tap on the supply pipe. It is discharged from the cistern through a siphon tube of uniform cross-sectional area s, the inlet to which is located near the base of the cistern, the arch of which is at a height A above the inlet and the outlet to which is at a distance B below the inlet, as shown in Fig. 1. Its discharge rate q will necessarily be variable. Until the water level in the cistern comes abreast of the summit of the siphon arch, q remains zero. When the siphon is functioning with the water level in the cistern momentarily at a height h above the siphon outlet, the speed of efflux u corresponding to this driving head h will, assuming ideal conditions, be given by $u^2=2gh$, so that $q=su=s(2gh)^{\frac{1}{2}}$.

The maximum discharge rate will correspond to the instant at which priming of the siphon occurs and will be given by $q_{A+B}=s[2g(A+B)]^{\frac{1}{2}}$. The minimum discharge rate occurs at the instant that the siphon inlet orifice becomes uncovered to admit air, thus arresting the siphoning action, and will be given by $q_B=s(2gB)^{\frac{1}{2}}$.

Clearly, when Q exceeds q_B, the emptying will never be complete; in other words, the siphoning process will be continuous. But if Q is less than this critical value q_B, the cistern will empty to the level of the siphon inlet, after which it will gradually refill until, in due course, the cycle of operations will recommence. This cycle comprises a storage phase alternating with a discharge phase. As the rate Q at which water is allowed to enter the cistern is normally very much smaller than the rate q_B at which the siphon is competent to secure its removal from the cistern, the discharge phase occupies an interval which is usually small in comparison with that occupied by the storage phase.

The period, including as it does the dual operation of filling and emptying, will depend (1) on the volume $V[=SA]$ of the cistern between the levels of the inlet and summit of the siphon tube, (2) on the dimensions of the siphon itself insofar as these influence the efflux rate q from it, and (3) on the value assigned to the rate of flow Q into the cistern. This period is arbitrary for it can evidently be controlled, within wide limits, by merely altering Q within the range $0>Q>q_B$. Furthermore, the oscillation could be

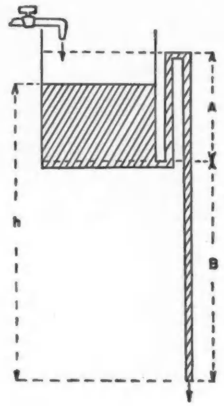

FIG. 1. Siphoning cistern.

temporarily suspended at any stage of the filling process simply by turning off the supply tap, and could be started at any subsequent instant from precisely the same point in the cycle by turning it on again. Such a system cannot in fact be said to exhibit any special predilection for any specific period.[1]

However, assuming Q to have a definite fixed value, the time taken to fill the cistern will be given by $T'=V/Q$. The time occupied in emptying it can readily be shown to be given by

$$T''=(S/s^2g)[(q_{A+B}-q_B)+Q\ln\{(q_{A+B}-Q)/(q_B-Q)\}].$$

The second term in square brackets is in the nature of a correcting term, arising because the influx into the cistern via the supply pipe persists throughout the siphoning stage. It is clear, in fact, that, if the supply tap is closed ($Q=0$) throughout the emptying interval, T'' will reduce to $(S/s^2g)(q_{A+B}-q_B)$, while as long as $Q\ll q_B$, this curtailed expression will still furnish a close approximation to the value of T''.

The orders of magnitude involved can best be appreciated by considering a representative quantitative example in which $A=25$ cm, $B=50$ cm, $S=500$ cm^2 and $s=(50)^{1/2}$ cm^2, so that $V=12{,}500$ cm^3, and $S/s^2g=10/g$. Hence $q_{A+B}=s[2g(A+B)]^{1/2}=10(75g)^{1/2}=2712$ cm^3 sec^{-1} and $q_B=s(2gB)^{1/2}=10(50g)^{1/2}=2215$ cm^3 sec^{-1}. Thus, if we take $T'=125$ sec, $Q=V/T'=100$ cm^3 sec^{-1} and $T''=(S/s^2g)[(q_{A+B}-q_B)+Q\ln\{(q_{A+B}-Q)/(q_B-Q)\}]=(10/g)497+(1000/g)\ln(2612/2115)=5.0_8+0.2_2=5.3$ sec. In view of the oversimplifying nature of the assumptions made, this cannot be expected to furnish anything more than a lower limit to the value of T''. Since, however, T'' amounts only to about 1/24 of T', or just over 4 percent of the whole period $T(=T'+T''\simeq 130$ sec$)$, its order of magnitude is all that matters. Indeed, for most practical purposes, T', by itself can be regarded as affording a sufficiently satisfactory approximation to the period.

It may be noted that the expression for T' involves the ratio of two physical magnitudes instead of the square root of such a ratio, as figures in the expression for the period of a harmonic oscillator. Moreover, the latter does not partake of the same arbitrary character. On the other hand, the amplitude A of the oscillation in question is fixed by the geometry of the siphon, or, more specifically, by the height of its arch above its inlet, whereas, in the harmonic oscillator, the amplitude is arbitrary within wide limits and, in the ordinary course of events, will be more or less rapidly extinguished by damping.

Thus, at the outset, we are presented with two striking points of contrast between the oscillation of the system under consideration and that of the familiar harmonic type. These alone would have justified the special name of *relaxation oscillation* conferred on the former by van der Pol. Actually, however, other distinguishing features of no less significance remain to be considered.

2. The self-actuating character of our simple hydromechanical contrivance enables it to provide a solution to a basic problem—that of maintaining a persistent oscillation from a continuous energy source. The siphoning cistern is supplied with energy at a constant rate by the fixed inflow trickle, and this energy is periodically released in the form of an intermittent discharge of water. The supply tap is necessarily located at, or above, the level of the siphon arch so that (assuming the water to emerge from the tap with negligible speed) the energy furnished to the system can be regarded as wholly potential. But since, during the storage phase, a volume V enters at a height at least $A+B$ above the outlet, the energy contributed to the system per period can be taken as equal to $SA\rho(A+B)g$, where ρ denotes the density of water.

The energy at the outlet during the discharge

[1] There is nothing inherently periodic about the operation of filling. It is the presence of the siphon which, acting as a triggering device, not only imposes a limit on the extent to which the filling can proceed but, at the same time, initiates and makes provision for the emptying of the cistern, thus insuring the regular repetition needed to confer a periodicity on this sequence of operations.

phase is wholly kinetic. If the water level at any instant is at a height h above the outlet the volume discharge for a decrease dh in h will be Sdh, and its kinetic energy will be given by $\frac{1}{2}Sdh\rho u^2$. Hence the energy recoverable from the system per period will be given by

$$S\rho g\int_B^{A+B} hdh = SA\rho(\tfrac{1}{2}A+B)g.$$

Thus a loss of energy amounting to $SA\rho(\frac{1}{2}A)g$ is inevitable and the maximum efficiency of the arrangement, measured by the ratio (maximum energy recoverable per period)/(total energy contributed per period), can be written in the form $1-\frac{1}{2}1/[1+(B/A)]$, from which it appears that $1>\text{Efficiency}>\frac{1}{2}$.

The efficiency would be unity if $A=0$, that is, if the supply from the tap spilled over directly into the siphon arch, thus virtually reducing the arrangement to a waterfall with a drop B. An alternative manner of achieving an equivalent result would be to set $Q=q_{A+B}$. However, both are irrelevant since the phenomenon would then be deprived of its periodic character.

The efficiency would be one-half if $B=0$, that is, if the siphon exit were on a level with its inlet. The larger B in comparison with A, the nearer the efficiency approaches to unity; and the smaller B in comparison with A, the closer it approximates to one-half. With $B/A=2$ (as in the example already cited), the maximum theoretical efficiency would be 5/6 or 83 percent.

Thus, though the efficiency of a harmonic oscillator may, theoretically, reach unity, that of the relaxation oscillator necessarily falls short of it.

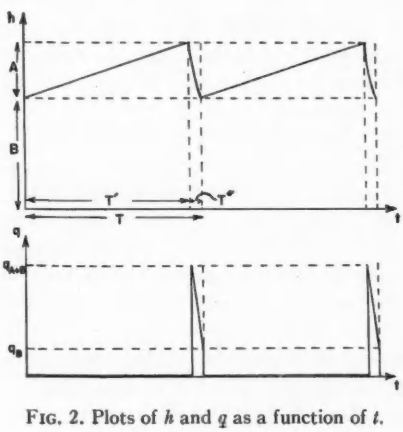

FIG. 2. Plots of h and q as a function of t.

3. Plots of h and q as a function of t are of the forms indicated in Fig. 2. The water level h in the cistern rises linearly with time from $t=0$ to $t=T'$, and then falls quasi-parabolically to its initial level at $t=T$. The discharge rate q is zero between $t=0$ and $t=T'$, jumps abruptly at the latter instant to the finite value q_{A+B}, then declines quasi-linearly, reaching the finite value q_B at the instant $t=T$, and thereupon drops instantaneously back to zero.

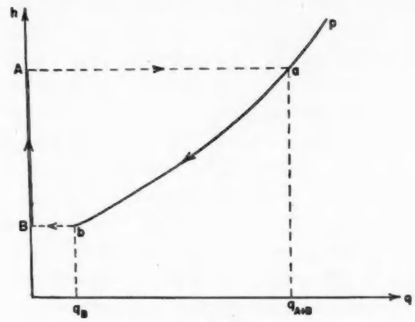

FIG. 3. Plot of h *vs.* q.

The comparative suddenness of the transitions from the storage to the discharge phase of the cycle, and vice versa, is responsible for the sharp kinks which are a feature of these curves. A "saw-tooth" wave profile of the kind exhibited by such an h-t curve implies, for the corresponding relaxation oscillation, a harmonic texture of considerable complexity as regards both the order and amplitude of its component terms.

The level h of the free surface of the water in the cistern above the siphon outlet represents what may be termed the "potential" variable of the system, while the discharge rate q from the siphon represents the corresponding "current" variable. A plot of h *versus* q, shown in Fig. 3, will accordingly depict graphically the "characteristic" of the system. It includes two distinct branches: one, coinciding with the segment BA of the h axis, corresponds to the storage phase (filling of the cistern with the siphon inoperative); the other, pab, corresponds to the discharge phase (emptying of the cistern by means of the siphon). If the inflow rate is such that the water level in

the cistern is consistently maintained at, or above, the summit of the siphon arch—that is, if $Q \geqslant q_{A+B}$—the outflow will be continuous at a rate determined by some point on the ap branch of the characteristic. When the oscillatory regime is established, the representative point will circulate indefinitely around the closed cycle $BAabB$. During the storage phase it traverses the vertical segment BA. At A, priming of the siphon is initiated, and an abrupt transition from $q=0$ to $q=q_{A+B}$ occurs, so that the tracing point jumps across from A to a on the same horizontal, but is located on the second branch of the characteristic. Along the latter, during the discharge phase, it descends along the arc ab and reverts abruptly, as soon as the siphon inlet is uncovered and the siphoning action temporarily suspended, to its starting point B.

The special property already mentioned, and common to all systems executing relaxation oscillations—ability to utilize energy from a nonperiodic source of supply to maintain themselves in periodic oscillation—results from the nonlinearity of their characteristic curves and the fact that the discharge rate (or, more generally, the current variable) is not uniquely specified by the head of water in the cistern (or, more generally, by the corresponding potential variable). The two branches of the characteristic curve are often connected by an arc of negative slope. We shall meet with an example of this in Part II. In the present instance this connecting link must be deemed imaginary.

The harmonic oscillator, unlike the relaxation oscillator, is unable directly to draw on a unidirectional source of supply to make good the inevitable frictional dissipation per cycle. The application of a constant external force will serve merely to displace the center about which the harmonic oscillations are performed; it will not, taken over a cycle, contribute any balance of energy to the system. To insure maintenance some device, capable both of partitioning out the energy available at the source and of synchronizing in appropriate phase relation its administration to the receptor, must be interposed between these two organs. The requisite conditions can be met by associating with the damped harmonic oscillator some suitable type of relaxation oscillator.

A timepiece is a case in point. The coiled spring, or raised weight, constitutes the potential energy store. The escapement controls the dosage of energy, and regulates its impulsive administration to the balance wheel, or pendulum. These impulses are delivered discontinuously, and are communicated to the harmonic oscillator at the instant when it is moving with maximum speed and the driving force differs in phase from the displacement by a quarter period, precisely when, in fact, their effect is a maximum in conserving its existing amplitude and a minimum in perturbing its natural period. Besides compensating for the decline in amplitude due to frictional causes, the escapement, through the scapement wheel, actuates the counting train, and so enables the passage of time to be registered on a dial, though this further function is immaterial in the present connection. The to-and-fro excursion of the anchor lever is limited by a pair of terminal stops and performs a typical movement of the relaxation oscillation class, though it may be noted that its action is dependent on its being coupled to the harmonic oscillator. Neither could continue to operate independently of the other. The escapement is needed to prevent the train racing and the motor impulses are essential to counteract damping agencies.

Even more readily accessible to observation is the mechanism embodied in the metronome, in which, as time intervals are reckoned aurally, it is possible to dispense with the counting train.

4. A harmonic oscillator can also be distinguished from a relaxation oscillator in respect of its behavior toward an imposed periodic force. If, to the former, an extraneous periodic force be applied, the system will oscillate in the period of the force with an amplitude that, in general, is small. Only if the forcing period happens to coincide with one of the natural periods of the harmonic oscillator does the amplitude of the forced oscillation become considerable. In other words, this condition of resonant response may manifest itself when the forcing frequency is attuned to the fundamental of the harmonic oscillator or to any member of its set of overtones, the frequencies of the latter, more often than not, bearing an integral relationship to the former. Though the frequency range of effective response is limited, the amplitude variation over it is normally pronounced, a small variation in the applied frequency in the vicinity of resonance producing far more than a proportionate reduction in the forced amplitude of the harmonic oscillator.

Nothing analogous to this enhancement of amplitude at resonance is to be anticipated with a relaxation oscillation, whose amplitude is fixed

by the level of the siphon arch above that of the siphon inlet when priming occurs, or, more generally, by the interval separating an upper and a lower critical value of the potential variable, the former initiating the discharge phase and the latter terminating it. On the other hand, in regard to range of frequency response, the relaxation oscillator is, as will appear, far more accommodating than the harmonic oscillator. When a forcing oscillation of period τ is imposed on one of the factors determining the period T of the relaxation oscillator and T happens to approximate to an integral multiple of τ, say, $p\tau$, the period of the relaxation oscillation is capable of automatically adjusting itself to assume precisely the value $p\tau$. In virtue of this property a relaxation oscillator can be locked into synchronism, not only with the applied forcing frequency, but also with any member of the set of integral submultiples thereof, or, if it is permissible so to phrase it, with its various subharmonic undertones.

By an appropriate combination of the frequency multiplying property of the harmonic oscillator with the frequency demultiplying property of the relaxation oscillator, it is possible, starting with a frequency ν, to derive therefrom a frequency given by $(p/p')\nu$, where p and p' are a pair of integers.

The two main factors governing the period of the relaxation oscillation under consideration are V (or, what is equivalent, h) and Q, and the simplest type of periodic variation that can be imposed on either is a sinusoidal one.

In the former case we may imagine the siphon arch fashioned like a trombone slide to enable a vertical oscillation of amplitude $h_0(<A)$ and period $\tau(<T)$ to be imposed on it, so that the water level in the cistern above the siphon outlet when priming occurs, being given by $h=H_0-h_0 \sin(2\pi t/\tau)$, may vary between H_0-h_0 and H_0+h_0. The level of the water in the cistern rises linearly with time but the amplitude of the corresponding relaxation oscillation may assume any value between H_0-h_0-B and H_0+h_0-B.

In the latter case we may imagine the supply tap controlled by a mechanism that causes the rate of delivery into the cistern from it to be given by $Q=Q_0[1-\cos(2\pi t/\tau)]$, where Q_0 represents the mean delivery rate. Though this method of procedure preserves unchanged the amplitude of the corresponding relaxation oscillation, the water level in the cistern no longer rises linearly with time.

We shall confine our discussion to the former alternative on account of the formal analogy it presents to the important practical problem of the linear time base employed in conjunction with the cathode-ray oscillograph.

The natural relaxation period, $T_{H_0+h_0}$, corresponding to the upper limiting priming level will clearly be larger than the natural relaxation period, $T_{H_0-h_0}$, corresponding to the lower limiting priming level. Moreover, the natural period T of the relaxation oscillation corresponding to an intermediate priming level H will be such that $T_{H_0+h_0}>T>T_{H_0-h_0}$.

As the discharge stage of the relaxation cycle takes proportionally longer from a higher level, refilling will start correspondingly later. Any pair of discharge curves will, however, descend parallel to one another below the lower of the two levels at which the respective discharges are initiated. Furthermore, as long as the rate of supply to the cistern is maintained unchanged, the rate at which the level of the water rises in it will remain constant and independent of the epoch at which refilling commences. Hence the lines representing the respective storage phases will run parallel to one another on an h-t diagram until they have risen to intersect the original priming levels.

The interval occupied over the storage stage of any particular relaxation cycle will, therefore, be directly proportional to the amplitude of the corresponding relaxation oscillation. Thus, reverting to our previous data, and taking H_0, the mean datum water level, as 70 cm, and h_0, the amplitude of its excursion, as 5 cm, we have Table I. By calculation, or from a plot of h against T, it is then possible to determine, over

TABLE I.

h (CM)	AMPLITUDE OF RELAXATION OSCILLATION ($h-B$, CM)	DISCHARGE PHASE T'' (SEC)	STORAGE PHASE T' (SEC)	PERIOD T ($T'+T''$, SEC)
75(H_0+h_0)	25	5	125	130($T_{H_0+h_0}$)
70(H_0)	20	4	100	104(T_{H_0})
65(H_0-h_0)	15	3	75	78($T_{H_0-h_0}$)

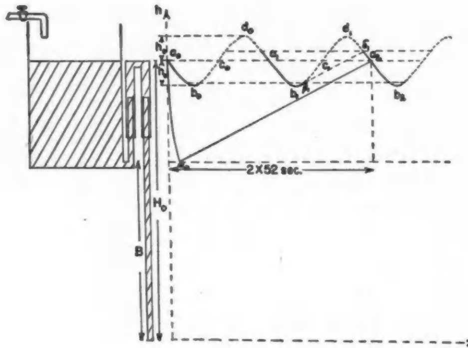

FIG. 4.

the range concerned, the value of h corresponding to a specified value of T, or vice versa; while the range covered could, if necessary, be expanded by increasing h_0. Thus, if T is 92 sec, then $h[=H]$ is 67.6 cm, so that the amplitude of the corresponding relaxation oscillation will be 17.6 cm.

Now suppose the magnitude assigned to the forcing period τ is such that some integral multiple of it, say, $p\tau$, satisfies the inequality $T_{H_0+h_0} > p\tau > T_{H_0-h_0}$. From the foregoing it follows that some priming level H can be found such that $H_0+h_0 > H > H_0-h_0$, and for which T has precisely the value $p\tau$. Once the appropriate phase relation between the relaxation oscillation and the forcing oscillation has been established to permit successive primings to occur at this level H, locking of the two oscillations in synchronism becomes theoretically possible.

Whether or not it is always practically realizable will depend on the relative values of the slope of the filling line and the slope of the tangent to the priming curve. The former is given by Q/S [since $h=(Q/S)t+B$], and has the value 0.20. The latter will be given by $dh/dt = -(2\pi h_0/\tau)\cos(2\pi t/\tau)$ so that when, say, $p=2$ and $\tau=52$ sec, this slope will assume its maximum value, $2\pi h_0/\tau = 0.60$, where the ascending arc of the priming curve crosses the mean datum level H_0. Hence, the slope of the line being less than the maximum slope of the tangent, a line can be drawn, as in Fig. 4, parallel to the filling line $\alpha_0 a_2$ to touch the priming curve at the point β_1, just past the trough b_1 on the ascending arc $b_1c_1d_1$. This line will intersect the priming curve in the point δ_1 somewhat beyond the crest d_1 on its descending arc $d_1a_2b_2$. Interlocking will clearly be prohibited at all points above this line, that is, throughout the arc $\beta_1c_1d_1\delta_1$, or any of the similarly situated arcs shown dotted in Fig. 4. The extent of the excluded arc will obviously increase with p; it would, for instance, include a larger fraction of the whole arc if $p=4$, $\tau=26$ sec.

We have been assuming here that, when successive primings occur at the mean level H_0, the period of the relaxation oscillation is exactly equal to double the imposed forcing period and, further, the two are so intercoupled that the initial priming takes place at the precise instant the descending arc of the priming curve crosses this mean level. However, as even over the available sections of the sinusoid it will not normally prove possible to couple up the relaxation oscillation in precisely the proper phase relation to the forcing oscillation, an interval of transition must, in general, precede synchronization.

Suppose, for example, that we take $\tau=46$ sec, so that, since $T_{H_0} > 2\times 46 > T_{H_0-h_0}$, the interlocking level will be located below the mean datum level H_0 (its actual value H being 67.6 cm). Let us assume, as before, that, at the instant of intercoupling (arbitrarily reckoned as the origin of time), priming takes place as the siphon arch, moving down, traverses the mean datum level H_0, corresponding to which the natural relaxation period would be 104 sec. But, as this exceeds 2τ, after two oscillations of the siphon slide, that is, at the epoch 2τ, when its arch is again passing down through the level H_0, the water in the cistern will not have risen as far as this level, so that priming cannot occur until the siphon arch has descended below it. The interval between the first and second priming will, accordingly, be greater than 2τ and less than T_{H_0}. Moreover, since the initial slope of the tangent to the discharge curve, given by $(dh/dt)_{h=H_0} = -s(2gH_0)^{\frac{1}{2}}/S$, has the value 5.2, while the slope of the tangent to the priming curve at this same level, H_0, has the smaller value 0.6, the discharge curve $a_0\alpha_0$, initiated from the point a_0 in Fig. 5, will lie consistently to the left of the priming curve a_0b_0. Hence the second priming level will be situated between the first

priming level H_0 and the interlocking priming level H.

It is a comparatively simple matter to calculate the levels of the water in the cistern at which successive primings occur. The second priming level H' is at 68.1 cm and is, therefore, as stated previously, such that $H_0 > H' > H$, while the interval between the first and second primings is 94.8 sec, which is both greater than 2τ and less than T_{H_0}. The third priming level H'' is at 67.8 cm, again such that $H' > H'' > H$, while the interval between the second and third primings is 92.6 sec, and so on.

Thus, subject to the prescribed conditions and starting from a priming level $H_0(>H)$, the interlocking level H will be approached asymptotically from above in the manner indicated in Fig. 5. Similarly, starting from a priming level lower than the interlocking level, the latter will be approached asymptotically from below. The more nearly comparable the slopes of the respective tangents to the discharge curve and the priming curve at the initial priming level, the more rapidly will the consecutive priming levels converge upon the interlocking level. For most practical purposes the approximation to the interlocking level can be regarded as terminating after a few periods have elapsed.

Moreover, synchronization, once established, will be stable, for should some chance perturbation cause priming to occur either a trifle prematurely, so that the discharge is initiated slightly above the interlocking level, or a little late, so that the discharge starts somewhat below this level, the next priming will, in both cases, occur between the previous priming and the interlocking level, and so nearer to the latter.

5. Many of the methods employed in maintaining vibrations in musical instruments afford illustrations of a mechanical type of relaxation oscillation.

In instruments of the violin family the hand-driven bow, moving throughout an up or a down stroke with sensibly uniform speed at right angles to the string, supplies the energy necessary to maintain the string in transverse harmonic vibration and makes good dissipation due to frictional causes and sound radiation. Its action depends upon the difference between the coefficients of friction when the bow grips the string and when it skids over it, which results from the fact that, whereas resin favors adhesion below a certain critical value of the relative speed, it functions as a lubricant for relative speeds above this critical value.[2] A similar example of more purely academic interest is the manner in which one can induce a stick of chalk to chatter by running it into the blackboard instead of propelling it over its surface in the normal fashion. With a little practice it is possible to develop a technic of drawing dotted lines expeditiously in this way.

In the flue mouthpiece a unidirectional air current, issuing under constant pressure into the atmosphere from a slitlike aperture (or "mouth"), is directed against a sharp edge (or "lip"), located vertically above, and parallel to, the slit. This moving sheet of air will be flanked on either side by a zone of turbulence separating it from the stationary, surrounding atmosphere. Incipient eddies are detached alternately from either side of, and with their cores parallel to, this slit orifice. In those originating from its right-hand edge the sense of circulation will be clockwise, and vice versa. These eddies travel at a speed which is necessarily inferior to that of the main air stream, while, in the course of their subsequent progress and development, their reaction on the central layer renders it sinuous. The presence of the upper lip at a fixed distance above the mouth converts this eddy sequence into a regular vortex system, inducing in it a spatial periodicity in which the interval separating consecutive vortexes in either zone of turbulence is equal to, or some integral submultiple of, the mouth height. It is the

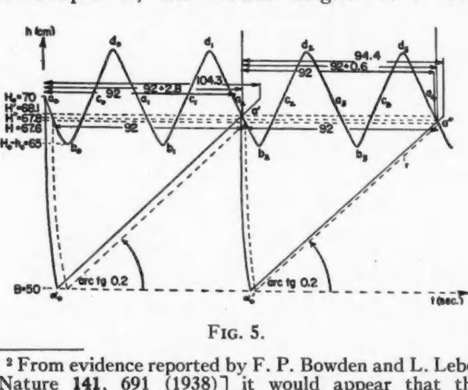

FIG. 5.

[2] From evidence reported by F. P. Bowden and L. Leben [Nature **141**, 691 (1938)] it would appear that this "stick" and "slide" process is fundamental in the interpretation of the phenomena of kinetic friction between solid surfaces. An interesting corollary to this view point is the finding that the friction between a piezoelectrically vibrating quartz surface and a stationary surface is reduced practically to zero [K. S. Van Dyke, Phys. Rev. **53**, 686 (1938)].

periodic passage of these vortexes alternately to right and left of the lip that gives rise to an edge-tone, whose frequency varies directly as the speed of efflux from the mouth, and inversely as the height of the lip above the mouth.

Commencing with the simplest condition of affairs, in which the core-to-core interspace along either vortex lane is equal to the mouth height, a progressive increase in the wind pressure causes the frequency of the free edge-tone to rise, until, at a certain critical pressure, it suddenly doubles. This jump-up of an octave in the pitch of the edge-tone occurs when the stable configuration for the allotted mouth height and the increased pressure corresponds to vortex lanes wherein the core-to-core interspace is reduced to one-half its previous value. In like manner, with the wind pressure held constant, an increase in the mouth height causes the edge-tone frequency to fall until a certain critical value is reached, whereupon a similar jump in pitch occurs.

An air cavity of appropriate form and dimensions in association with, and in the domain of, such a mouthpiece constitutes a flue pipe. If the wind pressure and mouth height are so arranged that the free edge-tone approximates in pitch to one of the partials of the air column, this edge-tone will be stabilized and reinforced by resonance. Though the reaction of the resonator on the source may not, under these circumstances, be very conspicuous, the note generated by such a coupled system differs to some extent from that of either of its separate components, and is peculiarly sensitive to pressure variations. If the pressure is reduced, so that the edge-tone frequency falls below that of the corresponding partial, the pipe note will drop in pitch, and vice versa. The range of response is restricted, and in order that the pipe may speak at all, the interval between the respective frequencies must not exceed a certain value—for example, a tone.

This must, however, be distinguished from the normal regime of functioning where a notably higher wind pressure is employed, and in which the edge-tone, obtained independently of the cooperation of the cavity by inserting a plug at the level of the pipe lip, is considerably (possibly several octaves) above the fundamental frequency with which the pipe speaks.

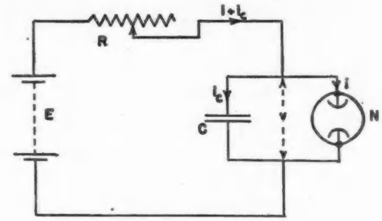

FIG. 6.

The periodic constitution of the air sheet in the vicinity of the mouth is analogous to that already described. An instantaneous glimpse of the mouth region, as afforded by an appropriate stroboscopic technic and admirably recorded in the aerograph designs of Carrière,[3] reveals the vortexes, in successive stages of organization from mouth to lip, staggered, after the fashion of lamp standards, with the lip extending like the yellow dividing line down the middle of the street. The passage of one of these vortexes along the inner surface of the lip, with the accompanying momentary diversion of the bulk of the air jet into the pipe, initiates a pulse of compression which travels up the pipe to be, in due course, returned down it, as a pulse of rarefaction if the end of the pipe is open, or as a pulse of compression if it is stopped. The arrival of the former at the mouth will tend to suck the jet into the pipe again and start a fresh pulse of compression up it, and vice versa. The periodicity of the mouth regime is thus dictated by the longitudinal vibrations induced in the air column. Their reaction on the source is of paramount importance, and synchronization is secured much in the manner already indicated. The pipe creates the note by imposing on the vortex system a periodic structure whose frequency is that of one of its partials. The pure edge-tone may itself subsist and be superimposed on the pipe note, but its contribution is a subsidiary matter. Though some variation of frequency with pressure does occur even here, it is much less pronounced than in the previous case, so that a flue pipe, with a suitably cut-up mouth operating in its normal regime, may tolerate a twelve-fold, or more, pressure-increase before yielding its next partial.[4]

[3] Carrière, J. de phys. et rad. **6**, 57 (1925).

[4] H. Bouasse, *Tuyaux et résonateurs* (Delagrave, 1929), chap. III; *Instruments à vent* (Delagrave, 1929), vol. I, chap. IV.

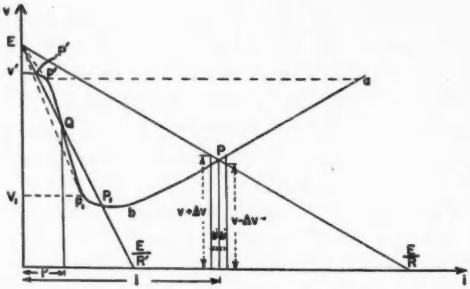

FIG. 7. Relationship of v to i for neon lamp.

Closely analogous phenomena are encountered in the humming of overhead telegraph lines and the whistling of wind in the cordage of ships. The frequency of the transverse frictional flutter induced as a result of the shedding of eddies alternately from either side of the wire is directly proportional to the wind speed, and inversely proportional to the wire diameter. Again no square root symbol is involved, as in the expression for the frequency of the ordinary transverse vibrations of a stretched wire. Aeolian tones are generated when the transverse forced vibrations, set up by this regular eddy detachment, approximate in frequency to some partial of the tensed wire.

Yet another example is provided in the heat-maintained sounding tube, the action of which depends upon the fact that, over a certain range, two alternative regimes of air-flow along the tube are possible, the one orderly and the other turbulent. The point at which the transition from one to the other takes place is conditional upon whether the approach is from the low speed or the high speed side. It is this factor that provides the special type of characteristic required to associate a definite periodicity with the unidirectional air-rise up the chimney stack.

II. ELECTRICAL

6. Electrical oscillations, essentially similar in character to those exhibited by the mechanical systems discussed in Part I, are obtainable from the simple circuit indicated in Fig. 6, where E is a battery serving as the unidirectional energy source, R is a suitable high resistance and C is a capacitance. At N there is a circuit element of the discharge tube type, exhibiting a specific ignition voltage V' and a specific extinction voltage V_1. It may conveniently be a neon glow lamp, which offers a practically infinite resistance so long as the potential difference v across its terminals remains below V'. If the emf of the battery exceeds V', the difference of potential v between the condenser plates builds up progressively until the value V' is attained. The tube then ignites, its resistance drops abruptly to a low value r and the condenser, being virtually short-circuited, discharges rapidly through it, dissipating energy in the form of light and heat, until the potential difference across its terminals falls to V_1, whereupon the discharge through the lamp ceases, its resistance again becomes practically infinite, and the cycle recommences. A knowledge of the characteristic, or $v-i$ relationship, pertaining to the discharge tube in question enables the phenomena to be interpreted. This characteristic for the neon lamp is of the form indicated in[5] Fig. 7.

Under static conditions, and with the condenser branch suppressed, the voltage v across the lamp terminals will be $E-Ri$, and depending upon where this line cuts the characteristic—that is, upon the value of R if E is constant—two distinct regimes of functioning are possible. If R is sufficiently small to insure intersection at some point, such as P, on the rising arc ba, the corresponding regime is stable and the lamp furnishes a constant illumination. For, suppose some momentary disturbance of the equilibrium conditions allows a current $i+\Delta i$ to traverse the lamp. The voltage $v-\Delta v$ available across its terminals being then, as Fig. 7 shows, below that required to sustain this increased current demand, the current will automatically decline to resume the value i, and equilibrium will be restored. On the other hand, should the current through the lamp momentarily fall to $i-\Delta i$, the voltage $v+\Delta v$ available across its terminals will more than suffice to maintain this current, which, accordingly, will increase to resume its original value i.

If R is sufficiently large to insure intersection at some point Q on the descending arc $V'b$, the corresponding regime is unstable. Any momentary increase in the value of i', the current corresponding to the point Q, will bring the representative point into a position for which the voltage available across the lamp terminals exceeds the value needed to maintain this current, which, accordingly, will tend still further to increase, and will, in fact, continue to increase until the point P_1 is reached. Similarly, any momentary decrease in i' will bring the

[5] Shaxby and Evans, Proc. Phys. Soc. **36**, 253 (1924).

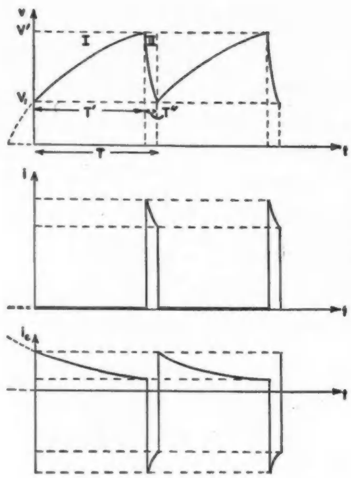

FIG. 8. Variation of v, i and i_c with time.

representative point into a position for which the voltage available across the lamp terminals is appropriate to an even smaller current. The current will, therefore, continue to decrease until the point P' is reached.

The slopes of the tangents drawn from the point E to touch the upper and lower extremities of the descending branch of the characteristic at the points p' and p_1 determine, numerically, a pair of values of R. If the magnitude of the external circuit resistance lies between the two limits thus defined, stable functioning of the lamp ceases to be permanently possible. It is replaced by a variable periodic regime in which, provided the repetition frequency is not unduly high, the representative point will trace out cyclically the path $V_1V'p'abp_1V_1$.

This same static characteristic can be regarded as applicable, at any rate as a first approximation, to the circuit of Fig. 6, which differs from that just considered only in that the condenser C is added in parallel with the neon lamp to supplement its self-capacitance. With the value of R adjusted to an appropriate figure, the voltage across the lamp and condenser builds up from V_1 to V' at a rate determined by the circuit parameters. When V' is reached the lamp starts to conduct, the representative point almost immediately arrives at the point p' beyond which instability sets in, and then flicks across, practically instantaneously, to a. At a the lamp strikes; the flash accompanying the partial discharge of the condenser is of short duration and persists until, just beyond b, the point p_1 is reached, at which instability again sets in, whereupon the representative point flicks across, again practically instantaneously, to V_1, and the cycle recommences.

The manner in which v, i and i_c vary with time is indicated in Fig. 8. During stage I, while the condenser is charging up from the battery through the resistance R, and the potential difference across its plates is rising from V_1 to V', the lamp resistance remains practically infinite, so that we may write $Ri_c+(1/C)\int i_c dt=E$, from which $i_c=\{(E-V_1)/R)\}\exp(-t/RC)$, so that the potential difference between the condenser plates will be given by

$$v=E-Ri_c=E[1-(1-V_1/E)\exp(-t/RC)].$$

This ascending arc of the $v-t$ curve, over which v increases, from the extinction voltage up to the ignition voltage, at a rate depending on the time constant appropriate to that portion of the circuit involved during the charging phase, corresponds to the V_1V' branch of the characteristic. Assuming E to be fixed, and treating V_1 and V' as specific constants for the particular neon lamp employed, the charging period will be given by $T'=RC\ln\{(E-V_1)/(E-V')\}$, which, accordingly, can be written in the form $T'=k'RC$, k' being a dimensionless constant. Subject to the supposition that any time lag in the establishment of the discharge is negligible, stage II, consisting of the discharge of the condenser through the lamp, can be considered to commence as soon as v reaches V'. The internal resistance of the lamp then drops from infinity to a value r, and, this latter being very small in comparison with the main circuit resistance R, the discharge will be practically confined to the lamp-condenser loop, in which the charged condenser constitutes the sole source of energy. This stage of the process will be governed by the equation $r(di_c/dt)+i_c/C=0$, from which, if r is regarded as constant (which, obviously, cannot be strictly true) $i_c=-(V'/r)\exp(-t/rC)$. The current through the lamp will be given by $i=-i_c=(V'/r)\exp(-t/rC)$, and the potential difference between the condenser plates by $v=(1/C)\int i_c dt=V'\exp(-t/rC)$.

The descending arc of the $v-t$ curve, over which v decreases, from the ignition voltage down to the extinction voltage, at a rate depending on the time constant appropriate to that portion of the circuit involved in the discharging phase, corresponds to the *ab*-branch of the characteristic. The discharging period will be given by $T''=rC \ln (V'/V_1)$, which, as before, may be written $T''=k''rC$, where k'' is another dimensionless constant subject to the same stipulation in respect of the invariability of V_1 and V'.

The period of the complete cycle is, therefore, $T=T'+T''=(k'R+k''r)C$, and includes two terms, each proportional to a time constant. The rhythmic potential fluctuation, produced by the alternate charging of the condenser through the resistance and its discharge through the lamp, constitutes a relaxation oscillation, electrical in nature, whose amplitude is fixed by an upper and lower critical voltage, and whose period is controlled by the circuit parameters determining the rate of supply of energy from the battery.

For many practical purposes it is legitimate to regard r as negligible in comparison with R, in which case we may write $k'RC$ as a first approximation to the period, and substitute for k', in place of $\ln \{(E-V_1)/(E-V')\}$, the more convenient approximate expression $(V'-V_1)/(E-V_m)$, where $V_m=\frac{1}{2}(V'+V_1)$.

7. An analogous type of circuit is employed in the linear time base, which has become an indispensable auxiliary to the cathode-ray oscillograph, except that the neon lamp is replaced by a thyratron, and the ohmic resistance R by the filament-plate interspace of a high vacuum diode operating under voltage saturation conditions. The modified circuit is shown in Fig. 9.

The thyratron is a hot cathode triode of special design, containing mercury vapor at a suitable

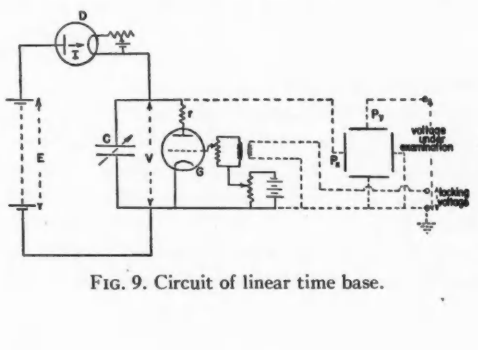

FIG. 9. Circuit of linear time base.

low pressure, and shares with the neon lamp the property of passing current only when the voltage across it attains a certain critical value. It, however, possesses certain advantages over the neon lamp. Provided its working temperature is maintained constant, the ignition and extinction voltages of the thyratron are less liable than are those of the neon lamp to progressive and erratic changes. Moreover, in the thyratron, the ignition voltage can be predetermined by the setting of the grid potential, so that, though the extinction voltage remains substantially independent of operating conditions, the voltage amplitude of the corresponding relaxation oscillation admits of considerable variation. If, for example, the grid voltage is set at the value $-V_g$, the thyratron will not ignite until its anode voltage attains the value kV_g, where k is a constant characteristic of the thyratron and termed its *grid control ratio*.

In the predischarge stage, when the conduction across the thyratron is negligible, the current I through the diode, which remains constant as long as its filament temperature is unchanged and the emf E of the battery is sufficiently high, will charge the condenser at a uniform rate, causing the difference of potential between its plates to rise linearly with time. In the circuit previously considered, where a constant resistance component takes the place of the constant current device, this can only be regarded as holding, and then only approximately, in the initial stage of the charging when the difference of potential developed across the condenser plates is a small fraction of E.

The interval occupied in increasing this difference of potential from V_1, the thyratron extinction voltage, to $V'(=kV_g)$, its ignition voltage, will be given by $T'=C(V'-V_1)/I$. But, in virtue of the circuit connections, the difference of potential across the condenser plates is applied between the cathode and anode of the thyratron, and, as soon as its value has risen to V', cumulative ionization will render the thyratron conducting. The positive ions generated effectively neutralize the space charge, thus allowing the full discharge to build up in an interval of the order of a few microseconds. This discharge current will be localized in the condenser-thyratron loop; and, to limit its value to the

current carrying capacity of the thyratron, a suitable safety resistance r must be included in series with the thyratron.

The grid is competent to serve as control element only during the predischarge stage. Once the discharge commences the grid is powerless to limit or extinguish it. Before the grid can reassume control the anode voltage must fall below the extinction value for a sufficient time to permit the ions present to disperse, and this, the so-called deionization time, may be of the order of 10μ sec.

If the effective resistance r of the discharge path is low, so that the current traversing the thyratron approaches its rated maximum value, which will certainly be large in comparison with I, the time taken for the difference of potential between the plates of the condenser to fall from V' to V_1 will be given, as before, to a close approximation by[6] $T''=rC \ln (V'/V_1)$.

The complete period will, accordingly, be $T=T'+T''$, and the wave profile of the corresponding relaxation oscillation will be of the "saw-tooth" form illustrated in Fig. 2. As T' is usually large in comparison with T'' the main portion of the period will be occupied in charging the condenser, and only the small remainder in discharging it.

Of the quantities which enter into the expressions for T' and T'', only V_1 and r are to be regarded as fixed. The relaxation period will thus depend upon the values assigned to the three remaining parameters, considered as possible variables. Two of these, V' and C, enter into the expressions for both T' and T'', while the other one, I, is involved in that for T' alone. If C and I be regarded, for the moment, as fixed, any variation in V', brought about by an appropriate modification of the bias imposed on the thyratron grid, will influence the period of the relaxation oscillation, through its amplitude, and so alter both T' and T''; while any variation in C or I (with V' unchanged) will influence the period independently of the amplitude. An alteration in the magnitude of C will affect both T' and T'' alike, since it will produce the same proportionate change in the time constants pertaining to both the charge and discharge circuits. Changing the value of I, by means of the diode filament rheostat will, on the other hand, affect T' exclusively.

This linear time-base circuit is coupled to the cathode-ray tube in the manner indicated in Fig. 9, so that the voltage variation developed between the condenser plates is applied across the vertical pair of oscillograph plates. If both of the horizontal pair of oscillograph plates are grounded, the luminous spot, resulting from the impact of the pencil of cathode rays on the fluorescent target, will travel horizontally across the viewing screen at a constant speed in one direction in a time T', and move much faster in the reverse direction over the same path in a time T''. The length of the base line will be determined, partly by the voltage-amplitude of the time-base relaxation oscillation, and partly by the stiffness of the cathode-ray beam.

In order to delineate, on the fluorescent screen, the displacement-time curve characterizing some particular periodic phenomenon, the latter, translated into corresponding voltage variations of appropriate amplitude by some suitable means, is applied across the horizontal pair of oscillograph plates. The electron beam will then be subjected to the joint action of the stated voltages simultaneously imposed across the two mutually perpendicular pairs of electrostatic deflecting plates. As, however, the frequency of the linear time-base will not, except by chance, coincide with that of the periodic phenomenon under investigation, or with any integral submultiple thereof, successive forward sweeps of the tracing pencil will, in general, commence at different points on the cycle representative of the phenomenon. This progressive phase shift will cause the trace to drift bodily across the viewing screen, the rate and direction of drift depending on the degree and sign of departure from integral relationship.

The ability to hold the pattern stationary on the screen, obviously convenient for visual study, and practically indispensable for photographic recording, is rendered possible by the extent to which the various parameters determining the time-base frequency are under the control of the operator.

Having secured a base line of suitable length, the use of the capacitance as coarse adjustment

[6] The unabbreviated expression is $rC \ln \{(V'-rI)/(V_1-rI)\}$.

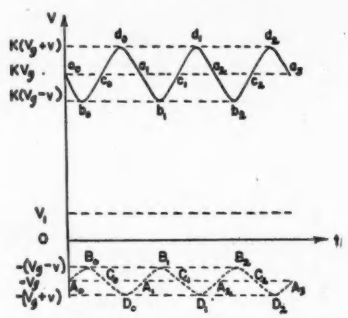

FIG. 10.

and the diode filament rheostat as fine adjustment will usually enable the time-base relaxation frequency to be brought into coincidence with the fundamental frequency, or some submultiple of the fundamental frequency, of the periodic phenomenon in question. Successive traces will then be accurately superimposed, and their resultant repetition will immobilize on the screen $p-1$ complete cycles of the phenomenon (where p is an integer) and the major part of the succeeding cycle. The remaining portion of the odd cycle will be accomplished during the much more rapid nonlinear flyback, and on that account, will be only faintly visible.

As long as the two voltages applied across the oscillograph plates are completely independent, manual adjustment can, at best, be relied on only to furnish a temporary stabilization of the pattern. Synchronization can, however, be rendered automatic without any very exacting requirements as to frequency uniformity, or any very precise preliminary tuning, by the simple expedient of feeding back a fraction of the voltage developed by this periodic phenomenon to the relaxation oscillator providing the time-base. This may be accomplished, as indicated in Fig. 9, by electromagnetic linkage between adjacent portions of the respective circuits. The control is particularly sensitive if the voltage variations picked up in this way are applied to the grid of the thyratron where an amplitude of a volt, or under, usually serves to lock the time-base in step, provided its frequency has been roughly tuned to that of the fundamental, or some selected subharmonic, of the periodic phenomenon under investigation.

Let us assume that the voltage fed back to the grid is given by $v \sin(2\pi t/\tau)$, where τ represents the period of the phenomenon to be recorded against the linear time base on the oscillograph screen. Then if, in the absence of this auxiliary voltage, the thyratron grid bias is set at $-V_g$, this auxiliary voltage will, when present, cause the grid bias to vary sinusoidally with time between the extreme values $-(V_g-v)$ and $-(V_g+v)$ with period τ. But, in view of the preceding remarks, it is legitimate to imagine this periodic voltage-variation transferred from the grid to the anode of the thyratron, provided its period is conserved, its phase reversed and its amplitude multiplied by the grid control ratio k. In other words, assuming its grid voltage to be given by $-[V_g-v\sin(2\pi t/\tau)]$, the corresponding ignition voltage will be given by $V'=k[V_g-v\sin(2\pi t/\tau)]$. We may, accordingly, confine our attention exclusively to the anode and regard the ignition voltage V' of the thyratron as varying sinusoidally between the extreme values $k(V_g-v)$ and $k(V_g+v)$ with the same period τ, in the manner indicated in Fig. 10.

Viewed in this light the problem of synchronization in the linear time base reduces, in essence, to that treated in Section 4, Part I, and, since the details have been discussed in another place,[7] they need not be repeated here.

8. The circuits considered so far include, in addition to the unidirectional source of electric energy and the current limiting device, a single reactance component, actually a capacitance, and a circuit element exhibiting a nonlinear characteristic. The latter acts as the control unit by imposing the potentials at which reversals occur. It is the presence of this that renders the circuit self-oscillatory, successive cycles comprising a progressive charge terminating in an abrupt discharge, and having a period expressible in terms of the time constants of the participating circuits. This simple arrangement can be regarded as a special case of a more general type of circuit. In the ordinary L, R, C series circuit the current i satisfies the equation $L(d^2i/dt^2)+R(di/dt)+(i/C)=0$, wherein, owing to the fact that energy is being continuously dis-

[7] Herrenden-Harker, Phil. Mag. **26**, Ser. 7, 193 (1938).

sipated in the resistance, the expression for i as a function of t must tend to zero as t tends to infinity.

If, however, there is substituted, in place of R, a component which, instead of resulting in a fall of potential proportional to the current traversing it, is capable of providing a rise of potential proportional to this current, the equation expressing the current in the L, $-R$, C series circuit will be $L(d^2i/dt^2)-R(di/dt)+(i/C)=0$. In virtue of the presence of what amounts, in effect, to a negative resistance, i should, in theory, increase indefinitely with t due to the continuous importation into the circuit of energy from the external source of supply. We have already encountered examples of circuit elements exhibiting the required negative pseudo-resistance. Others include the triode, intercoupled in such a manner as to permit feedback of energy from its anode to its grid, the dynatron, a triode whose grid is maintained at a higher potential than its anode and in which secondary emission causes a decrease in current with increase of voltage, and the most venerable of all, the carbon arc.[8]

The rise of potential across the terminals of all such practical contrivances can, however, be regarded as proportional to the current traversing them only over a more or less restricted range; some factor or other, sooner or later, intervenes to prevent unlimited energy drainage from the source. What happens in fact is that, though the effective resistance of the device is initially negative, its actual value is dependent on the magnitude of the current, the functional relationship being such as, eventually, to insure that the effective resistance of the device becomes positive for currents exceeding a certain magnitude. Thus, while the amplitude of an incipient oscillation will build up at the outset, an oscillation whose amplitude surpasses a certain critical value will be damped down in the normal manner. In between, an amplitude can be found such

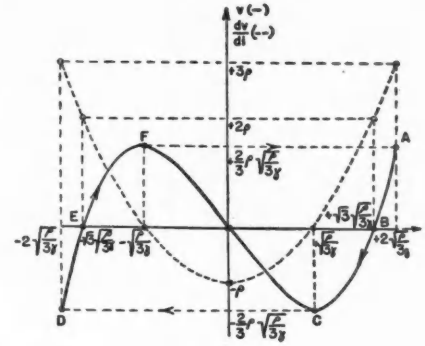

FIG. 11.

that the energy furnished to the system during a certain fraction of the cycle precisely balances the energy dissipated during the remainder of the cycle, and to which corresponds a stable regime of persistent sustained oscillation.

Imagine, therefore, included in series with the inductance L and capacitance C, a more general type of circuit component whose characteristic is expressible by the relation $v=f(i)$. Its equivalent resistance $R[=dv/di=f'(i)]$ will thus no longer be either a positive or a negative constant but a certain function of the current traversing it, so that, in a variable regime, R will depend on time.

The current i in this circuit will satisfy the equation $L(di/dt)+v+(q/C)=0$, or $L(d^2i/dt^2)+f'(i)(di/dt)+(i/C)=0$. The simplest type of characteristic that will automatically secure the limitation of current which is imperative on physical grounds is a cubic one, whereon the component in question is set to operate in the vicinity of a point of inflection. It is, moreover, of considerable practical importance since it is applicable to the regenerative triode oscillator.

Writing, therefore, $v=f(i)=-\rho i+\gamma i^3$, where ρ and γ are positive constants, the characteristic will be of the form indicated by the full line of Fig. 11. The dotted line in the same figure represents $f'(i)=-\rho+3\gamma i^2$, which, as it appears, is negative for values of i included between $(\rho/3\gamma)^{\frac{1}{2}}$ and $-(\rho/3\gamma)^{\frac{1}{2}}$. It is the bends in the characteristic at C and F that operate as a brake on an initially increasing amplitude, and lead to a final stable oscillation of finite amplitude.

[8] Circuits in which these components figure will obviously be capable of generating relaxation oscillations. The complaint known colloquially as "motor boating," which may manifest itself as a low frequency throb in the output of an amplifier, is of this type since it does not depend on the resonant frequency characteristic of, but on the time constant pertaining to, the particular circuit involved. Usually it is discouraged by decoupling, but, in the multivibrator, which is essentially a symmetrically disposed two-stage back-connected resistance-capacitance-coupled amplifier, it is encouraged and exploited.

Our basic equation accordingly becomes $L(d^2i/dt^2)+\rho[-1+(3\gamma/\rho)i^2](di/dt)+(i/C)=0$. This may be put into a more convenient form for subsequent treatment by the introduction of two new dimensionless variables x and y, expressible in terms of t and i by the relations $x=t/(LC)^{\frac{1}{2}}=t/t_0$ and $y=i/(\rho/3\gamma)^{\frac{1}{2}}=i/i_0$, respectively. It may be noted that i_0 is the value of i which annuls the quantity appearing in square brackets in the foregoing equation, leaving $L(d^2i_0/dt^2)+(i_0/C)=0$, the equation characterizing the simple undamped circuit whose natural period is $T_0=2\pi(LC)^{\frac{1}{2}}$, so that $t_0=T_0/2\pi$. By this artifice the original equation can be made to assume the "reduced" form $(d^2y/dx^2)+\epsilon(-1+y^2)\times(dy/dx)+y=0$, containing only the single parameter $\epsilon=\rho(C/L)^{\frac{1}{2}}$, itself a positive dimensionless number.

When $\epsilon=0$ the term involving dy/dx drops out leaving $(d^2y/dx^2)+y=0$, which represents a simple harmonic oscillation of unit pulsatance, that is, of period 2π. For values of ϵ other than zero the only solutions so far available, even for this especially simple type of nonlinear characteristic, are based on the somewhat tedious methods of graphical integration developed by van der Pol,[9] Lienhard[10] and Usui.[11, 12] Without attempting to outline these methods, for details of which reference should be made to the sources indicated, certain broad conclusions can be reached from quite elementary considerations.

It is apparent at once that, for values of y so small that their squares can be neglected in comparison with unity, the equation becomes $(d^2y/dx^2)-\epsilon(dy/dx)+y=0$, which represents a harmonic oscillation, possessed of a positive increment, the amplitude of which will, at any rate in the initial stages, build up exponentially. Such a state of affairs can, from the nature of the problem, be only temporary; and, as soon as y^2 exceeds unity, the increment is transformed into a decrement, that is, an actual damping. The limiting form of the oscillation, and the interval required to establish it, will clearly depend upon the value assigned to ϵ.

If $\epsilon\ll 1$, so that terms in ϵ^2 can be neglected, we may write $y=X+\epsilon X_1$. Substitution of this in the reduced form of the original equation, and retention of only first powers in ϵ, gives $(d^2X/dx^2)+\epsilon(d^2X_1/dx^2)+\epsilon(-1+X^2)(dX/dx)+X+\epsilon X_1=0$, which yields the pair of relations

$$(d^2X/dx^2)+X=0,$$
$$(d^2X_1/dx^2)+(-1+X^2)(dX/dx)+X_1=0.$$

Since our trial solution relates only to the permanent steady state, at which the system arrives after a finite interval of transition, we can arbitrarily assume that $X=X_0$ when $x=0$. The solution of the first of the foregoing pair of equations will then be $X=X_0\cos x$, and substitution of this in the second gives $(d^2X_1/dx^2)+X_1=-X_0(1-X_0^2/4)\sin x+(X_0^3/4)\sin 3x$. A trial substitution of $X_1=X_{01}\sin 3x$ leads to the relation

$$-9X_{01}\sin 3x+X_{01}\sin 3x$$
$$=-X_0(1-X_0^2/4)\sin x+(X_0^3/4)\sin 3x,$$

to satisfy which we must have $X_0=2$, since $X_0=0$ is irrelevant, and $-8X_{01}=X_0^3/4$, that is, $X_{01}=-\frac{1}{4}$. Thus, finally, $y=2\cos x-(\epsilon/4)\sin 3x$, or, as under these conditions the fundamental will be the preponderating term, we may write as a first approximation, $y=2\cos x$.

The final permanent state will, therefore, be an approximately sinusoidal oscillation, the fundamental amplitude of which is 2 and the fundamental period 2π. Since, in terms of i and t, this equation becomes $i=2(\rho/3\gamma)^{\frac{1}{2}}\cos 2\pi\times\{t/2\pi(LC)^{\frac{1}{2}}\}$, the corresponding expressions for the amplitude and period will be $A_{\sin}=2(\rho/3\gamma)^{\frac{1}{2}}$, and $T_{\sin}=2\pi(LC)^{\frac{1}{2}}$. In virtue of the small value of the negative damping coefficient $-\epsilon$, and the initial insignificance of the y^2-term, it is clear that this limiting amplitude will be attained only after an interval comprising a relatively large number of periods, during which the amplitude will build up exponentially.

The opposite condition, $\epsilon\gg 1$, transforms the oscillation from the sinusoidal into the relax-

[9] van der Pol, Phil. Mag. **2**, Ser. 7, 978 (1926).
[10] Lienhard, Revue Générale de l'Electricité **23**, 901, 946 (1928).
[11] Usui, *Report of radio research in Japan* **5**, 39 (1935).
[12] The present unsatisfactory situation in regard to this important problem has prompted the issue of a recent appeal to mathematicians to bestir themselves to attempt the formulation of an explicit analytic solution for these nonlinear differential equations. Though the analytic difficulties in the general case are doubtless formidable, it is surely not unreasonable to anticipate that mathematical technics claimed to be competent to grapple with the Universe might be turned to account in solving the more mundane perplexities that arise in connection with the humble triode.

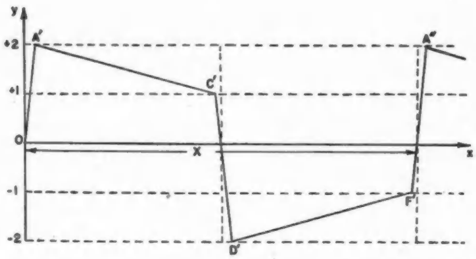

FIG. 12. The interval marked x is the period $\mathfrak{X}$.

ational type, the ultimate stationary state of which will correspond to repeated circulation of the representative tracing point around the contour $ABCDEFA$ of the characteristic depicted in Fig. 11. This tracing point will flick over the linear segments CD and FA in a time which is small in comparison with that occupied in traversing the arcs ABC and DEF. Since the values of $y[=i/(\rho/3\gamma)^{\frac{1}{2}}]$ for the various points on the characteristic are

Point	A	B	C	D	E	F	A
Value of y	2	$\sqrt{3}$	1	-2	$-\sqrt{3}$	-1	2,

the corresponding relaxation oscillation will be of the form indicated in Fig. 12, the ordinate of the $y-x$ curve falling slowly from 2 to 1, then dropping abruptly from 1 to -2 to rise, at first slowly, in the second half of the period from -2 to -1 and thereafter rapidly from -1 back to 2.

The arcs $A'C'$, $D'F'$, which occupy the major portion of the period, being quasi-linear will have a finite slope and an infinitesimal curvature. If, therefore, it is legitimate to neglect the term d^2y/dx^2 in comparison with the others the reduced equation becomes $\epsilon(-1+y^2)(dy/dx)+y=0$, which can be integrated immediately, giving $x=\epsilon(\ln y-\frac{1}{2}y^2)+\text{const}$. Since the quasi-vertical arcs $C'D'$, $F'A''$ account for only a negligible fraction of the whole period $\mathfrak{X}$, we may write, as a first approximation, $\mathfrak{X}=2\epsilon|\ln y-\frac{1}{2}y^2|_2^1=\epsilon(3-\ln 4)=1.61\epsilon$. The final state, in this case, is thus an oscillation whose amplitude $A_{rel}[=2(\rho/3\gamma)^{\frac{1}{2}}]$ is the same as in the previous case, but having a period given by $T_{rel}=1.61\epsilon t_0=1.61\rho(C/L)^{\frac{1}{2}}(LC)^{\frac{1}{2}}=1.61C\rho$, which, again is expressed in terms of the time constant pertaining to the participating circuit. When, at the outset, the term in y^2 can be neglected, the large negative value of the damping coefficient $-\epsilon$ renders the circuit aperiodic. Accordingly, as the initial value $y=0$ is unstable, the transient interval preceding the attainment of the permanent steady state will consist in an exponential departure of y from zero unaccompanied by oscillations. The final stationary amplitude is, in point of fact, practically established after an interval of the order of a single period of the relaxation oscillation.

These largely intuitive conclusions in regard to the nature of the solutions of the basic equation $(d^2y/dx^2)+\epsilon(-1+y^2)(dy/dx)+y=0$ for the selected pair of values of ϵ receive confirmation in Fig. 13, wherein van der Pol has exhibited graphically the relation between y and x corresponding to the three values $\epsilon=0.1$, $\epsilon=1$ and $\epsilon=10$. The first and third of these practically realize the two extreme cases already discussed. The dotted curves in the latter, which are plots of $\ln y-(y^2/2)=(x/\epsilon)+\text{const.}$, with appropriately adjusted values assigned to the constant, are seen to approximate satisfactorily to the actual solution over the ranges $y=\pm 2$ to $y=\pm 1$. The value $\epsilon=1$ corresponds to an intermediate type of oscillation which fairly rapidly builds up to its terminal amplitude, while the final profile, attained after an interval comprising relatively few periods, affords evidence of a distinct departure from the simple sinusoid.

The manner in which the oscillation passes over progressively from the sinusoidal to the relaxational type as ϵ is continuously increased from a value in the vicinity of zero can be traced experimentally with a dynatron shunted across an LC-circuit, in which the magnitude of C is arranged to be gradually varied.

III. Biological

9. The fate of a particular biological community in a prescribed habitat, as evidenced by the vital statistics relating to it, will be governed by a variety of factors, including prevailing climate, available means of sustenance, degree and kind of social organization, etc.

Consider, by way of preliminary, a population consisting of a single species and including n individuals at the epoch t. The difference Δn between this number and that expressing the population figure at the later epoch $t+\Delta t$ will be accounted for by the fact that, during the

interval Δt, certain individuals will have been added to the population by birth or immigration, while others will have been removed from it by death or emigration. If the population is reasonably large and not too highly dispersed, and if the time elapsing between the two censuses is not too great, the number of births and deaths will be jointly proportional to n and to Δt so that we may write for the former, $\beta n\Delta t$, and for the latter, $\delta n\Delta t$. Though the numbers of immigrants and emigrants will likewise both be directly proportional to Δt, they will bear a less simple relation to n. Denoting them, provisionally, by $I\Delta t$ and $E\Delta t$, respectively, we have $\Delta n=\beta n\Delta t-\delta n\Delta t+I\Delta t-E\Delta t$. Hence the mean time rate of variation of the population will be given by $\Delta n/\Delta t=\beta n-\delta n+I-E$, so that, subject to the limitations imposed, which are necessary in virtue of the essentially discrete character of n, we may write, in the limit, $dn/dt=\beta n-\delta n+I-E$.

As we shall be confining our attention exclusively in what follows to closed populations we may put $I=E=0$, and so dismiss the possibility of contributions from either of these sources at the outset.

In general it is not legitimate to assume that β and δ are constant. It is obvious, for instance, that as the concentration of the population rises the pressure exerted by it on its habitat will be intensified. The mutual rivalry among its members for the available means of sustenance will become progressively more acute, which, in turn will result in a birth rate that decreases and a death rate that increases with the figure of the population. The simplest manner of taking quantitative account of such trends is to assume their dependence on n to be linear, and to write

$$\beta=\beta_0-\beta_1 n,\quad \delta=\delta_0+\delta_1 n.$$

The preceding equation then becomes $dn/dt=(\beta_0-\delta_0)n-(\beta_1+\delta_1)n^2$, or, more compactly, $dn/dt=\mu n-\lambda n^2$, where μ is a coefficient symbolizing, when $\beta_0>\delta_0$, the inherent capacity of the population to expand, or, when $\beta_0<\delta_0$, its intrinsic inability to survive; and λ is a coefficient of limitation symbolizing the adverse influence of population pressure on the habitat, which operates to check the former tendency and to hasten the latter.

This equation, written in the form $(dn/n)-dn/(n-n\lambda^{-1})=\mu dt$, gives, on integration, $n/(n-\mu\lambda^{-1})=\exp(\mu t+\kappa)$ or, if $n=n_0$ when $t=0$, $n=-(\mu/\lambda)n_0/[(n_0-\mu\lambda^{-1})\exp(-\mu t)-n_0]$. It is clear that, when t is sufficiently large to render the exponential term insignificant, the population figure will become stationary at the value $n=n_m=\mu/\lambda$. The curve representing the growth of the population with time is S-shaped, starting from $n=n_0$ when $t=0$ and approaching the saturation level $n=n_m$ asymptotically.

On the other hand, if λ is so small that the quadratic terms can be omitted, $n=n_0\exp(\mu t)$; in other words, if $\beta_0>\delta_0$, the population will increase exponentially, while, if $\beta_0<\delta_0$, it will decline exponentially

10. Results more relevant to our present purpose arise out of a theoretical study of the simultaneous development, within a circumscribed environment, of a pair of coexisting and

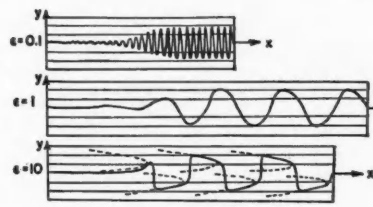

FIG. 13. [After van der Pol.]

interacting living species—one of which may be animal and the other vegetable, or both may be of either kind. Lotka[13] has treated this problem from the standpoint of the now fashionable method of "biologic control" of an insect colony by the importation of a strain of parasites to undermine its constitution. Volterra[14] has considered it from the point of view of the evolution of two varieties of fish, one preying upon the other in a landlocked sea. Both are basically analogous problems since if, without intent to dogmatize biologically, we agree to distinguish our fishes as sharks and soles, the former can be regarded as large-sized parasites with hostile intentions toward the latter. The point of particular interest is that, subject to certain con-

[13] Lotka, *Elements of physical biology* (Williams & Wilkins, 1925), p. 88.

[14] Volterra, *Leçons sur la théorie mathématique de la lutte pour la vie* (Gauthier Villars, 1931), chap. I, sec. 2.

ditions, which may be realized at least approximately in practice, the mathematical analysis furnishes a periodic solution thus indicating that both species can continue indefinitely in existence side by side, their numbers exhibiting regular alternating expansions and contractions.

At the epoch t let n_1 be the number of the host, or edible, population and n_2 the number of the parasite, or predatory, population. The former will be faced with a supplementary risk of decease due to the parasitic, or predacious propensities of the latter. The number of encounters between the members of either will be proportional to the product of the numbers of each present, and, since this will evidently vanish with either n_1 or n_2 separately, can be written as $\nu n_1 n_2$. Assuming each such encounter to prove fatal to the member of the first population figuring in it, we may write as our first equation, $dn_1/dt=\mu_1 n_1-\lambda_1 n_1^2-\nu n_1 n_2$.

Consider next the factors affecting the evolution of the second population. If its members are parasites upon the first, or host, population we may suppose each parasite to lay, on the body of the host to which it gains access, a number of eggs, p of which will, on an average, hatch out to develop, in due course, into adult parasites, meanwhile encompassing the death of its host. Thus $p\nu n_1 n_2$ can be taken as representing the normal rate of multiplication of the parasites. If δ_{02} denotes their natural death rate, and λ_2 the coefficient of limitation arising from mutual competition between members of the parasite population, our second equation can be written as $dn_2/dt=-\delta_{02}n_2-\lambda_2 n_2^2+p\nu n_1 n_2$.

If the second population consists of sharks preying upon the first, or sole, population the available means of sustenance of the former will depend directly and exclusively on the number of the latter. Should, for instance, n_1 diminish, the birth rate of the sharks will decrease while their death rate will increase, and vice versa. Hence, both the vital coefficients, previously denoted by β and δ, must, before they can be regarded as applicable to the second population, have included in them terms proportional to n_1. Accordingly we may write

$$\beta_2=\beta_{02}-\beta_{12}n_2+\beta_2' n_1,$$
$$\delta_2=\delta_{02}+\delta_{12}n_2-\delta_2' n_1,$$

which will give as our second equation,

$$dn_2/dt=\beta_2 n_2-\delta_2 n_2$$
$$=(\beta_{02}-\delta_{02})n_2-(\beta_{12}+\delta_{12})n_2^2+(\beta_2'+\delta_2')n_1 n_2.$$

In view of the fact that, should the first population become extinct, the second population will inevitably be doomed to extinction we can assume δ_{02} to exceed β_{02} and write $\beta_{02}-\delta_{02}=-\mu_2$, when $dn_2/dt=-\mu_2 n_2-\lambda_2 n_2^2+\nu' n_1 n_2$.

Thus both problems lead to essentially similar types of equation, which, on the further assumption that the concentrations of both populations remain sufficiently low to render mutual competition between the members of either of negligible importance, reduce to the somewhat more tractable forms

$$dn_1/dt=n_1(\mu_1-\nu n_2),$$
$$dn_2/dt=n_2(-\mu_2+\nu' n_1),$$

wherein all the coefficients are positive and are assumed to be constant.

These equations may be put in the form $dn_1/n_1(\mu_1-\nu n_2)=dn_2/n_2(-\mu_2+\nu' n_1)=dt$, which, on multiplication through by the product of the two parenthesis terms, gives

$$(-\mu_2+\nu' n_1)dn_1/n_1=(\mu_1-\nu n_2)dn_2/n_2$$
$$=(\mu_1-\nu n_2)(-\mu_2+\nu' n_1)dt. \quad (1)$$

Accordingly, if we denote $[n_1]_{t=t_0}$ and $[n_2]_{t=t_0}$ by n_{10} and n_{20}, respectively, integration of Eq. (1) will give

$$\int_{n_{10}}^{n_1}\left(\nu'-\frac{\mu_2}{n_1}\right)dn_1=\int_{n_{20}}^{n_2}\left(-\nu+\frac{\mu_1}{n_2}\right)dn_2$$
$$=\int_{t_0}^{t}(\mu_1-\nu n_2)(-\mu_2+\nu' n_1)dt,$$

from the first pair of which we obtain

$$\nu'(n_1-n_{10})-\mu_2 \ln n_1/n_{10}$$
$$=-\nu(n_2-n_{20})+\mu_1 \ln (n_2/n_{20}),$$

or

$$n_1^{\mu_2}n_2^{\mu_1}\exp(-\nu' n_1-\nu n_2)=K_0$$
$$=n_{10}^{\mu_2}n_{20}^{\mu_1}\exp(-\nu' n_{10}-\nu n_{20}), \quad (2)$$

K_0 being a constant whose value is determined when the initial conditions, that is, the values of n_{10} and n_{20}, are specified.

Equation (2), which may be written in the alter-

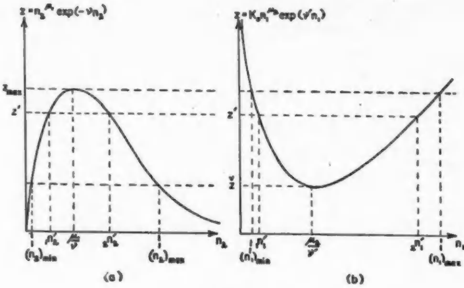

FIG. 14.

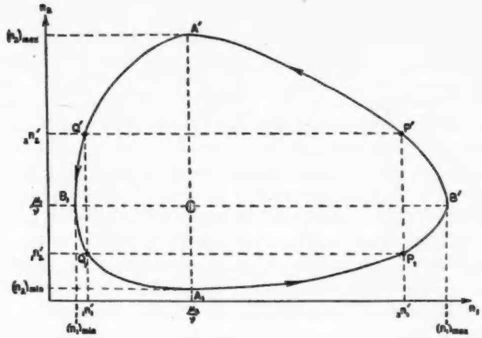

FIG. 15.

native form, $n_2^{\mu_1} \exp(-\nu n_2) = K_0 n_1^{-\mu_2} \exp(\nu' n_1)$, will represent a curve in the n_2n_1-plane, the general character of which can be arrived at from a preliminary consideration of the pair of auxiliary curves

$$Z = n_2^{\mu_1} \exp(-\nu n_2),$$
$$Z = K_0 n_1^{-\mu_2} \exp(\nu' n_1).$$

In the former the ordinate Z starting from zero when n_2 is zero will increase with n_2, pass through a maximum value $Z_{\max}$ when $n_2 = \mu_1/\nu$, and thereafter decrease, eventually approaching the axis of abscissa asymptotically. The plot of Z against n_2 will, accordingly, be of the form shown in Fig. 14(a). In the latter the ordinate Z, starting from infinity when n_1 is zero, will decrease, pass through a minimum value $Z_{\min}$ when $n_1 = \mu_2/\nu'$, and subsequently increase, tending ultimately to infinity with n_1. The plot of Z against n_1 will accordingly be of the form shown in Fig. 14(b). A horizontal line through $Z_{\max}$ will thus be a tangent to curve (a) and intersect curve (b), provided $Z_{\max} > Z_{\min}$, in a pair of points the abscissas of which are $(n_1)_{\min}$ and $(n_1)_{\max}$. No real value of n_2 can be found to correspond to values of n_1 less than $(n_1)_{\min}$ or greater than $(n_1)_{\max}$.

A horizontal line through $Z_{\min}$ will intersect curve (a) in a pair of points the abscissas of which are $(n_2)_{\min}$, $(n_2)_{\max}$, and be a tangent to curve (b). No real value of n_1 can be found to which will correspond values of n_2 less than $(n_2)_{\min}$ or greater than $(n_2)_{\max}$. A horizontal line drawn through any point Z', such that $Z_{\max} > Z' > Z_{\min}$, will intersect each of the two curves in a pair of separate points. Thus to each of the values ${}_1n_1'$, ${}_2n_1'$ of n_1 will correspond the pair of values ${}_1n_2'$, ${}_2n_2'$ of n_2. With the data secured in this manner it now becomes possible to plot out, step by step, the n_2n_1-curve which must of necessity be a closed one similar to that indicated in Fig. 15.

The fact that this curve is closed implies that both n_2 and n_1 are periodic functions of t. Each circulation of the representative tracing point in a counterclockwise sense once around its perimeter, with a speed varying from point to point along it, will occupy an interval equal to, or advance the time by, one complete period T. Starting out, say, at the epoch t_0 from the point A_1, where the number of the host population is $n_{10}(=\mu_2/\nu')$, the corresponding number of the parasites $n_{20}[=(n_2)_{\min}]$ is so reduced as to occasion little inconvenience to their hosts, who, accordingly, increase rapidly in number, displacing the representative point toward the right along the arc A_1P_1. Such conditions are ideal for the multiplication of the parasites, but their prosperity can be only at the expense of the hosts, whose numbers, after a pseudo-pause in the vicinity of B', commence to decline. At a point such as P' the host population is perceptibly waning while the parasite population continues to expand. Some time later, in the region of A', the hosts have become so depleted in numbers that the parasites start to fade out rapidly due to the shortage of their natural food supply. When the point B_1 is passed, and the parasites have been largely wiped out, the host population starts to develop anew, which brings us back to the point A_1, at the epoch t_0+T, and starts the same cycle over again.

Though the present resources of mathematics seem unable to furnish any formal analytic solution for the period from the original pair of nonlinear differential equations, the periodic nature of the variation in the respective numbers of the two populations with time must be similar to that indicated in Fig. 16. Moreover, both systematic statistics relating to the fish populations of the Adriatic and data derived from studies of entomophagous parasites tend to confirm the prediction of a periodicity of this type. Though the period T is common to both populations the maximum of the first population occurs at an earlier epoch in the cycle than does the maximum of the second population, so that the cyclic variation in the first, or host, population exhibits a certain phase advance with respect to that of the second, or parasite, population.

In spite of the fact that a natural cyclic process of this sort differs somewhat from those already discussed in Parts I and II, it bears sufficient formal resemblance to them to merit its inclusion in the category of relaxation oscillations. Once again we encounter an example of a phenomenon, in itself essentially aperiodic, that can, under appropriate circumstances, be converted into one capable of repeating itself indefinitely and automatically at regular intervals. Neither population, in independent occupation of the habitat, could exhibit any such rhythmic variation. The numbers in the one would dwindle to extinction, those in the other would expand to saturation. The fact that the members of one population prey exclusively upon those of the other when both share the same habitat confers on the numbers of either their periodic character.

A different set of initial conditions will necessitate the substitution of K_0' in the place of K_0 as the constant of integration. If K_0' exceeds K_0 the ordinates of Fig. 14(b) will have to be expanded in the ratio K_0'/K_0. It is not difficult to see that, to this augmented value K_0' of the constant of integration, will correspond a closed curve in the n_2n_1-plane which is smaller than, and completely enclosed by, that depicted in Fig. 15, which corresponds to the value K_0. Similarly, if K_0' is less than K_0, the corresponding closed curve will be larger than, and will completely encircle, the original curve. For each of these closed curves the maximum and minimum values of n_2 will be located on the line $n_1=\mu_2/\nu'$, while the maximum and minimum values of n_1 will be situated on the line $n_2=\mu_1/\nu$. The point C, whose coordinates are μ_2/ν', μ_1/ν, can, therefore, be regarded as a

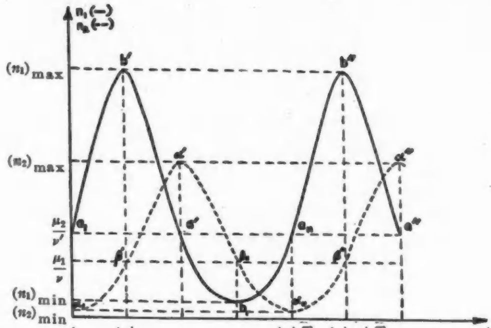

FIG. 16.

common center for each member of the family of curves. It is, moreover, itself a member of this family, that for which the ratio K_0'/K_0 is such as to make $Z_{\min}=Z_{\max}$. Its coordinates represent the mean values, extended over a complete period, of the numbers of the respective populations.

In effect, if we integrate our original pair of equations over a period we obtain, since

$$\int_{t_0}^{t_0+T}(dn_1/dt)dt=\int_{n_{10}}^{n_{10}}dn_1=0$$

and

$$\int_{t_0}^{t_0+T}(dn_2/dt)dt=0,$$

the two relations

$$\mu_1\int_{t_0}^{t_0+T}n_1dt=\nu\int_{t_0}^{t_0+T}n_1n_2dt,$$

$$\mu_2\int_{t_0}^{t_0+T}n_2dt=\nu'\int_{t_0}^{t_0+T}n_1n_2dt.$$

If, likewise, we integrate, over a period, these same equations, written in the alternative form $(-\mu_2+\nu'n_1)dn_1/n_1=(\mu_1-\nu n_2)dn_2/n_2=(\mu_1-\nu n_2)(-\mu_2+\nu'n_1)dt$ we obtain

$$0=-\mu_1\mu_2T+\mu_1\nu'\int_{t_0}^{t_0+T}n_1dt$$
$$+\mu_2\nu\int_{t_0}^{t_0+T}n_2dt-\nu\nu'\int_{t_0}^{t_0+T}n_1n_2dt,$$

so that, since

$$\mu_2\nu\int_{t_0}^{t_0+T}n_2dt=\nu\nu'\int_{t_0}^{t_0+T}n_1n_2dt$$

in virtue of the second of the two previous relations, it follows that

$$\overline{n_1}=(1/T)\int_{t_0}^{t_0+T}n_1dt=\mu_2/\nu'.$$

Similarly, as

$$\mu_1\nu'\int_{t_0}^{t_0+T}n_1dt=\nu\nu'\int_{t_0}^{t_0+T}n_1n_2dt,$$

it follows that

$$\overline{n_2}=(1/T)\int_{t_0}^{t_0+T}n_2dt=\mu_1/\nu.$$

It is clear that these means, about which the respective population numbers fluctuate, will conserve their values irrespective of the initial conditions.

In view of the importance of the relation which this point C bears to the population cycle, our equations can be made to assume a more significant form if we transfer the origin to this common center by substituting $N_1+\bar{n}_1$ for n_1 and $N_2+\bar{n}_2$ for n_2. In terms of these new variables, N_1N_2, representing the respective excesses of each population above its mean figure, our fundamental equations become

$$dN_1/dt = -\nu N_2[N_1+(\mu_2/\nu')],$$

$$dN_2/dt = \nu' N_1[N_2+(\mu_1/\nu)],$$

which, despite their enhanced significance, appear to be equally intractable.

Suppose, however, that, for some reason or other, the natural death rate of both species is increased, involving the substitution of $\mu_1-\Delta\mu_1$ for μ_1 in the first of our original pair of equations and of $\mu_2+\Delta\mu_2$ for μ_2 in the second. Then, if T' is the period of transcription of the cycle corresponding to the equations thus modified, the respective population means become

$$\bar{n}_1' = (1/T')\int_{t_0}^{t_0+T'} n_1 dt = (\mu_2+\Delta\mu_2)/\nu',$$

$$\bar{n}_2' = (1/T')\int_{t_0}^{t_0+T'} n_2 dt = (\mu_1-\Delta\mu_1)/\nu.$$

Hence, as Volterra showed, an increment in the mortality of the host population results in a lowering of the mean number of the parasite population, while an increment in the mortality of the parasite population leads to a raising of the mean number of the host population.

Fishing, directed primarily against the marketable species, is equivalent to a supplementary cause of mortality among the sole population, and as Volterra has also pointed out, the trawling away of a certain proportion of the host (or sole) population, provided it is carried out at the appropriate epoch in its cycle of development, when, normally, the sharks would be flourishing at its expense, will result in an augmented catch of soles in the next and succeeding cycles—an observation of considerable moment to the fishers if they could be persuaded to put their trust in mathematical pronouncements.

In this connection it must not be forgotten that such conclusions as we have arrived at are subject to certain simplifying assumptions, adopted in order to clear the equations of second degree terms in n_1 and n_2. The mathematical mill, however, moves so protestingly in these matters that such further complications, introducing additional grit into the material fed to it, are calculated to arrest its operation completely.

Though, under the rather special conditions postulated, the parasite population can never, theoretically, succeed in exterminating the host species, it may in actual practice so deplete its numbers as to expose it to other inimical contingencies, ignored in our analysis, which will render its continued existence decidedly precarious. In fact, any natural or artificial cause that interferes with the existing conditions is liable to upset the delicate dynamical balance between the two populations and may lead to a succession of cycles, which, instead of being exactly superposable in the n_2n_1-plane, spiral in toward C. The solution, though it may conserve its periodic character, will then be damped instead of persistent, or, alternatively, it may cease to be strictly periodic and become what Kostitzin[15] calls *periodomorphic*.

Certain involuntary physiological processes, such as the beating of the heart and the chattering of teeth, are essentially relaxational in nature. In fact, van der Pol and van der Mark[16] have devised an electrical model of the heart capable of furnishing electrocardiograms reproducing not only the normal but also certain well-recognized abnormal heart actions by coupling together, in a specific manner, three relaxation circuits.

The primary nervous excitation, consisting of some form of electrical impulse propagated along a nerve, provokes a reaction that is independent of the intensity of the excitation and of the "all or nothing" variety. This, likewise, has its counterpart in the behavior of a relaxation system, which, when subjected to an impulsive stimulus, either does not respond at all, or, if the moment of its delivery is so timed that the system is approaching its periodically recurring unstable state, responds fully with an amplitude independent of the magnitude of the triggering impulse.

Plant life over the greater part of the surface of the globe is normally exposed to a diurnal cycle in which day alternates with night. The

[15] Kostitzin, *Biologie mathématique* (A. Colin, Paris, 1931), p. 30.

[16] van der Pol and van der Mark, Phil. Mag. **6**, Ser. 7, 763 (1928).

leaves and blossoms of certain plants exhibit what are termed nyctinastic movements in synchronism with the solar illumination. These rhythmic movements appear to be an inherent property of the plant, since they continue to manifest themselves, with a period in the neighborhood of 24 hr, even when the plant is kept in the dark. It has further been found that, when a plant of this sort (*canavalia*) is subjected to the influence of an artificial periodic illumination having a fundamental period of 6 hr, its leaf movements automatically synchronize with the fourth subharmonic of the imposed illumination —another instance of the frequency demultiplication so typical of relaxation oscillations.

It is also probable that the periodic outbreaks of epidemics, and the recurrence of economic crises, may be ascribable to an underlying mechanism of an essentially relaxational kind. At all events current evidence goes to prove that a slump occurs with far greater rapidity than the succeeding climb up the prosperity slope to the boom peak.

In fact, it seems more than likely that fuller investigation will lead to the interpretation of an increasing number of naturally occurring periodic processes along "relaxational" lines.

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Measurement in Physics

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RECENT methodological critiques of quantum mechanics have had much to say about measurement and its place in the theory. It is probable that a good deal of this discussion has been rather unintelligible to the average physicist and to his colleagues in philosophy and in the other sciences who are interested in knowing what the modern atomic physicist thinks he is doing. This prompts a re-examination of certain aspects of the meaning of measurement in physics, particularly in connection with the microscopic domain. It is, of course, not possible within the limits of a brief article to do justice to the subject in all its phases.

The first experiments in physics, like those in any field of human activity, were restricted to the establishment of qualitative correlations between observations—the dropped stone was observed to fall, the red glow of the metal in the fire was associated with its heat, the suspended magnet was observed to assume a definite preferred orientation with respect to the earth's meridian, etc. But physicists have rarely been contented with this level of the description of nature. One of the curious developments in man's psychological evolution is the appearance of the urge toward quantitative description. In Greek science in particular this can be traced back to the Pythagorean doctrine that nothing in the universe can be adequately discussed without the use of number. The origin of the number concept itself is so shrouded in the mists of antiquity as to render surprising the fact that it did not make an even earlier decisive impact on the development of scientific method. That it did not is very likely due to its limited usefulness until experimentation had reached a more mature level. Of course, measurement as a practical human activity is of very ancient origin, as the measurement of land, the weighing of money and the estimation of time from the observation of the heavenly bodies all attest. Curiously enough, however, a long time elapsed before the quantitative aspects of these practical, economic operations were con-

sidered of value in the scientific description of natural phenomena.

Measurement in the modern physical sense arose concomitantly with the increasing sophistication of experiment, that is, with the idea of an experiment as a set of carefully prearranged operations with more or less elaborate apparatus. Even the time of appearance of this development is uncertain. There is no question, however, that we meet it definitely in the work of Archimedes, greatest of the physicists of antiquity. Not only did he speculate about the property of the lever and the behavior of bodies immersed in fluids on the basis of mathematical postulates, but he actually constructed apparatus and verified his deductions experimentally. This is particularly well illustrated in his solution of the famous problem of Hieron's crown, where he was able to reduce the ratio of the gold and silver in the crown to a ratio of volume differences. His measurement of solid volumes by fluid displacement was a stroke of genius. The reduction of the solution to volume measurement was naturally characteristic of a geometer who was accustomed to deal with area and volumes metrically. Significant with respect to Archimedes' use of numbers in describing the results of his experiments is the fact that he threw numerical quantities into the form of ratios or pure numbers. Most of his famous geometric theorems on volumes and areas were expressed in this way. It was natural that in his physical experimentation he should carry over the mathematical technic previously found to be effective. A similar situation is encountered in Archimedes' law of the lever in which the ratio of weights appears as the ratio of distances from the fulcrum. This was derived by mathematical reasoning from intuitive assumptions, but there is every reason to believe from contemporary accounts that Archimedes actually used the law to construct machines having a previously predicted mechanical advantage. The success of mathematical reasoning applied to such practical purposes must have acted as a great stimulus toward the quantitative description of experiments and so contributed greatly to the devising of new and more accurate methods of assigning numerical values to such fundamental quantities as length, time and weight.

Much has been written by physical methodologists about the nature of measurement. About all they agree on is that measurement fundamentally is the association of numerical values with physical observations. To be sure, there is also general agreement that not *every* association of numerical values with an observation constitutes a measurement: certain criteria must be imposed. Thus N. Campbell[1] insists that measurement consists of an operation in which a numerical value is assigned to a *property* of a physical body or system, and that a property in order to be measurable must be *additive*. This means that if the value of the property for one system is a and that for another system is also a, the value of the property of the resultant system composed of both systems is greater than a. For example, weight satisfies this criterion while density obviously does not. Campbell, therefore, does not consider density to be a directly measurable property. However, since methods and instruments exist by which a numerical value *can* be associated with the density of a system by means of a single operation—for example, use of the hydrometer—the value of the distinction as a fundamental criterion appears rather questionable without more searching inquiry. There is involved, of course, the well-known distinction between *extensive* properties (for example, weight) and *intensive* properties (for example, density).

In an interesting article on the methodology of quantum mechanics, E. Schrödinger[2] suggests repeatability as the fundamental criterion for a measurement: an operation on a system will be called a measurement of some property characteristic of the system if the same numerical result reproduces itself within certain allowed limits of error on the immediate repetition of the operation, it being assumed that no extraneous influences have come into play in the meantime. This, of course, raises the whole question of the part that errors play in measurement, but most physicists probably would accept the criterion as reasonable. However, it is not our purpose at this point to make a searching critique of such criteria in themselves but rather to point out that both those just mentioned involve an

[1] Campbell, *Physics—the elements* (Cambridge Univ. Press, 1920).

[2] Schrödinger, Naturwiss. **23**, 807, 823, 844 (1935).

interesting underlying assumption; namely, that there exist things which we call objects and that these objects have sharply defined properties, the assignment of numerical values to which is the province of measurement. This clearly illustrates the growth of what may be called the *object-instrument idea* along with the development of more elaborate physical theories. According to this idea, measurement amounts to taking an appropriate instrument and applying it to the object, with the aim of attaching a numerical value to some property of this object. It is, perhaps, a good thing to point out that this idea, natural enough from the standpoint of everyday affairs where the number of properties of objects of interest is relatively few and the instruments are of such ancient origin that we accept their operation uncritically and almost unconsciously, nevertheless, involves a considerable amount of convention when applied to more complicated physical operations, particularly those of the modern physical laboratory. Let us therefore examine a little more carefully the meaning of the object-instrument idea from the standpoint of what a physicist actually does when he makes a measurement.

For the sake of illustration, consider the measurement of *temperature*. In the first place, we have the purely qualitative consideration that one object feels hotter than another, that one season is warmer than another. This is succeeded by the scientific feeling that, if such a situation is to be described adequately, we must replace the crude estimate of human feeling by something more reliable and reproducible, something that employs the visual sense rather than the tactual, since human beings have come to trust the former as being definitely more trustworthy than the latter. We observe that many visual changes take place in objects during heating. Thus, recall that Galileo fashioned a glass bulb with a tube of small bore attached to it and arranged the apparatus vertically with the open end of the tube just beneath the surface of some colored liquid in a dish. When the air in the bulb is warmed by the hand it expands and bubbles through the liquid. When the bulb is thereafter cooled the liquid rises in the tube. The level of the liquid in the tube as read on a scale placed alongside was used by Galileo as a measure of the temperature of the air in the bulb. To calibrate the instrument it was necessary only to assign arbitrary numerical values to the readings on the scale for two well-known physical conditions of the bulb, for example, its immersion in melting ice and in steam under atmospheric pressure. Now it is important to realize that the process just described is not merely a method of measuring temperature; it is really a *definition* of temperature. If this were the only thermometer in use, whenever we employed the symbol t in any relation to which numerical significance is attached we would have to associate with it the values read from the scale on this instrument. On the purely operational level the same situation is encountered in all thermometers, whether based on the expansion of substances with heating, the change in resistance of electrical conductors, the thermoelectric effect, etc. Each of these supplies a separate definition of temperature, and we need not expect that any two of these thermometers will agree precisely in their readings for the same environment even if they have been calibrated in the same way. As a matter of fact they do not agree, although over certain useful ranges the discrepancies are often small. This is an encouraging circumstance, since it suggests the possibility of a universal definition of temperature in terms of a *theory* of heat. It is the desire to treat the concept of temperature as a theoretical construct which is responsible for the view that the measurement of temperature is the process of placing a certain *instrument* called a thermometer in the neighborhood of an *object* having a certain property (or better, state) which it is the function of the thermometer to measure. Before the introduction of this theoretical construct there is little excuse for saying more than that the operation with a particular instrument permits assignment of a number to a quantity which we call temperature and which is observed to vary from one environment to another; if we term this "the measurement of temperature," it is then fairest to say that the thermometer measures its own temperature.

This purely empirical attitude is, of course, altered by the introduction of a theoretical construct, in which temperature is viewed as a variable descriptive of the state of a physical system that is itself a model constructed in

terms of the theory of mechanics. The precise definition of temperature then depends on the type of model used; that is, whether it is of the macroscopic thermodynamic or the microscopic kinetic theory variety. In both cases the concept of temperature is closely tied to that of the mechanical concept of energy. In any event, the construct has too much theoretical utility for it to be dispensed with, and the endeavor is made to produce a consistent connection between it and the experimental measurement. This association is itself carried out by means of a model, namely, the ideal gas, since the theory shows that for a constant-volume thermometer employing such a gas the empirical definition agrees with the theoretical definition. Of course, there exists no actual gas having the properties of an ideal gas, but the theorist is not dismayed by this fact and proceeds to devise a theoretical scheme whereby the temperature measurements made with an actual constant-volume gas thermometer can be translated into the temperature values consistent with the theory. At every step of this ingenious process the attention of the physicist is focused on the theoretical model, and it is small wonder that he should gradually come to think of the experimental measurement of temperature as being in every respect merely the assignment of numerical values to a property or state of the physical object of which the model is the theoretical construction.

The procedure just described in the case of temperature finds its counterpart in every branch of physics and has come to be the accepted interpretation of measurement. Its success has been enhanced by the fact that theoretically deduced properties of the model have suggested new experiments establishing new relations among already well-known quantities. Now physical theories employ constructs of two different kinds: (1) constructs more or less immediately suggested by experience and whose corresponding symbols enter directly into the physical laws which are the immediate description of laboratory experiments; and (2) more abstract constructs to whose symbols numerical values cannot be assigned directly by any actual measurement. Thus, the position of the center of mass of a body, and its mass, velocity and the time are illustrations of the first kind of construct. In the second category it might be supposed that we intend to place the constants or coefficients that occur in physical laws, such as the moduli of elasticity, etc. This is not, however, what we have in mind, though it is true that such constants gain their numerical values only through the direct measurement of other quantities occurring in the same equation. Rather, for class (2) we have in mind the purely abstract constructs that characterize, for example, the classical wave theory of light and modern atomic theory, where we imagine entities that are never observed in order to account for certain phenomena that *are* observed. Thus, in the wave theory of light, physicists have not hesitated to introduce the concept of *displacement* of a purely hypothetical light-bearing medium, which corresponds to no directly measurable quantity. Nevertheless, we believe that it is an extremely useful concept, since it facilitates the derivation of equations connecting quantities which *are* measurable and so enables the construction of a coherent theory of optical phenomena. The tendency toward the use of such abstract concepts has grown steadily with the advance of theoretical physics, so that in modern atomic theory, for example, we are perfectly reconciled to the use of constructs like the position and velocity of an electron in an atom. This again is justified by the fact that by building a theory based on these concepts we can ultimately derive equations connecting quantities which possess laboratory significance. If these equations or "laws" of atomic theory, when tested, are verified, the theory is satisfactory; and more particularly so if the theory is in a position to make predictions as to the existence of hitherto unsuspected laws. This is now standard procedure; and few physicists object to it in principle, though some have undoubtedly viewed with reluctance the apparently growing tendency to use more abstract concepts in the construction of theories.

The use of constructs of the second kind in physical theorizing, does, however, necessitate a certain degree of caution. It will be noted that many, if not most, of these constructs are logically well defined with reference to macroscopic effects; for example, we think we know what we are talking about when we speak of the position and displacement of a large-scale par-

ticle, for we have laboratory methods by which to measure them. It is not surprising, therefore, that we are tempted to use similar concepts for entities which have only hypothetical existence. This is a natural and legitimate method of procedure as long as the use of such concepts eventually leads to actual laboratory equations, in other words, makes the theory produce some sort of identification of the abstract constructs with measurable quantities. There is a danger, however, that after much preoccupation the abstract construct will grow to seem so real that we try to give it direct meaning in terms of experimental measurement by the invention of imaginary mental experiments. Even this would not be a particularly questionable procedure if the mental experiments were used merely as a guide to the suggestion of new actual experimental measurements. If, however, they are taken seriously as a definite criterion of what measurements are really possible and under what restrictions actual measurements may take place, trouble seems bound to ensue. It is perfectly possible that the mental measurement in question may be a valid deduction from the theory, *if* the measurement could be carried out. But the question at once arises: Of what value is its validity if the measurement cannot be carried out except "in principle"? It is likely to give an exaggerated importance to an element of the theory that, after all, may not be very significant.

It is important to recognize that all physical theories overpredict experience in the sense that they contain elements which do not correspond with anything we actually observe. This state of affairs is usually glossed over with little or no comment in classical physics, presumably because we feel that we know what we are doing anyway and are perfectly competent to pick out and use those parts of a theory which correspond to experience without needing to worry too much about those which transcend it. Consider such a comparatively simple mechanical problem as the damped harmonic oscillator. In classical mechanics the parameter t is supposed to be able to assume both positive and negative values. The resulting equations are descriptive of experience for nondissipative systems, but the attempt to employ negative values for t in the case of dissipative systems leads to results having no counterpart in experience. Yet we do not on this account discard the usual simple theory of dissipative systems as of no value. We merely introduce a restriction in its use, which amounts to an interpretation as to how the theory is to be used. We are careful in this case not to attach undue significance to the result of applying negative values of t to purely idealized systems and to the resultant theoretical conclusions that might be drawn. Physicists have not considered this a useful procedure.

The classical physicist generally was not seriously embarrassed by the overpredictive tendency of physical theories. The contemporary physicist, however, has been inclined to lay too much stress on it and, in consequence, has often puzzled and bewildered the layman and especially his philosophical colleague. A good illustration is provided by the theory of relativity. This theory makes certain predictions that can be tested experimentally. Actual physical measurements appear to be in substantial agreement with these predictions, for example, the measurements which we describe in terms of the variation of the mass of charged particles with velocity. On the other hand, the theory also makes predictions about the results of measurements that hypothetical observers might make on ideal systems. The latter are all in the class of mental measurements and have probably done more to confuse the lay public, and even experimental physicists themselves, as to the meaning of the theory than any of the mathematical technics necessary for its development. Insofar as these purely imaginary deductions—for example, concerning observers traveling with the velocity of light—are regarded merely as exercises to increase the student's grasp of the technic involved in the theory, somewhat like the fearful and wonderful mathematical exercises to be found in the older English textbooks of mechanics, valid objection cannot be raised to them. But when they are taken as a serious basis for a philosophical evaluation of the *meaning* of the theory, we cannot help feeling that a distorted view of physics and physical measurement is bound to emerge.

The serious use of mental experiments and hypothetical measurements has reached its high watermark in the quantum theory of atomic structure. It is, of course, well known that

several different mathematical formulations of the theory, of varying degrees of elaboration, are possible. Although all these formulations may lead with reasonable interpretation to the same actual experimental results, it is not unlikely that they may differ in what they have to say about hypothetical, idealized situations having no counterpart in experience. This makes it questionable to take seriously, for example, the criticism of Einstein, Podolsky and Rosen[3] who claim to find logical difficulties in quantum mechanics by an analysis of "measurements" performed on ideal systems. The authors reach the disconcerting conclusion that the state of one such system completely isolated from another system nevertheless depends on the type of measurement performed on the latter system. Much attention has been paid to this problem. No one questions the validity of the reasoning, but the interpretation of the result has been criticized by a number of writers. It seems rather curious, however, that in every critique the conclusion of hypothetical or imaginary experiments is taken just as seriously as if the measurements actually could be performed. This is a rather good indication of the thoroughgoing fashion in which theoretical physicists tend to attribute complete reality to every element of a theory and in particular to transfer bodily the object-instrument point of view of experimental physics to the abstract constructions of the theory. If the discussion were restricted to actual experimental measurements, the difficulty would disappear or, better, it could not have arisen at all. This sort of trouble is inherent in the tendency to pay too serious attention to the overprediction of theory. In this connection we should note the interesting fact that Margenau[4] has shown that the dropping of a single postulate from the most common formulation of quantum mechanics—the one employed by Einstein, Podolsky and Rosen—makes the difficulty vanish without impairing the ability of the theory to predict the actual experimental data described by the theory. This in itself is good evidence of the danger involved in attributing real experimental significance to too many elements of a theory by analyzing the performance of "measurements" on systems that cannot be realized experimentally. Let us again emphasize that this procedure is not necessarily to be deprecated *per se*. But its justification, if any, must come only with direct experimental verification. It is not reasonable to draw fundamental conclusions from "measurements" that cannot actually be performed.

Much of the current discussion of the principle of indeterminacy in quantum mechanics falls into the pitfall just described, with its analysis of the "measurement" of position and velocity of an electron by purely mental methods. So long as this is used merely as a heuristic device for lending picturesqueness to an abstract theory, no objection can be raised. But one must protest when it is used to draw far-reaching conclusions as to what kinds of measurement are fundamentally possible and as to basic limitations on the possibility of measurement in general. The unsuspecting layman and even the experimental physicist are pretty sure to interpret these conclusions as referring to actual measurements performed in the laboratory, whereas the "measurements" in question are only those performed in the mind of the theorist with the elements of the theory.

To summarize the essential points of this brief critique, the attempt has been made to show how the object-instrument point of view concerning measurement, which is a perfectly natural outgrowth of the development of theoretical physics, leads to the danger that in modern physical theorizing too much emphasis is laid on purely mental measurements. When we consider the overpredictive tendency of all physical theories this is seen to be a rather unfortunate extrapolation which tends to place unreasonable restrictions on the possibility of measurement as commonly understood by physicists and leads to confusion. It is contended that any theory of measurement must ultimately concern itself with measurements that can actually be performed in the laboratory and not with merely hypothetical mental "measurements" on idealized systems.

[3] Einstein, Podolsky and Rosen, Phys. Rev. **47**, 777 (1935).

[4] Margenau, Phys. Rev. **49**, 240 (1936).

The Temperature Concept

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In physics texts temperature is ordinarily treated as a quantity which, like mass, length and time, is fundamental for problems and discussions involving heat. On the other hand, there are those who declare that temperature is just a convenient term for the average translational kinetic energy of a molecule of the body under consideration, or, as some prefer to state it, the energy per mole associated with the translatory motion of its molecules. Whether or not temperature or, for that matter, some other quantity in the field of heat involving temperature may be viewed in this or some similar manner and expressed in terms of the basic mechanical quantities mentioned, is the question under consideration here.

If the temperature of a body is merely a measure of the average translational kinetic energy of the molecules of the body, we should be able to write

$$T = k\cdot \tfrac{1}{2}m(v^2)_{Av}. \qquad (1)$$

If we should select *on this basis* a temperature unit that would fit in consistently with other units of the cgs system, k would automatically become unity. Accordingly, we would say that, when the average translational kinetic energy of the molecules of a body is one erg per molecule, the body has a temperature of one erg. The temperature of some hydrogen, say, at the melting point of ice thus expressed in ergs would then be $5.66\cdot 10^{-14}$ erg. Obviously the erg would be an inconveniently large unit of temperature, and one can understand why it would be desirable to take $2.07\cdot 10^{-16}$ erg as one centigrade degree.

If physics were strictly classical, the foregoing identification of temperature with translational energy per molecule might reasonably stand. In the field of the classically ideal gas, the policy of identification does not give rise to difficulty. But gases, even ideal gases in the region of absolute zero where they become degenerate, are not classical; and liquids and solids are much more nonclassical. Where quantum numbers are small, the distribution of energy among the molecules of a body is very different from the classical expectation.

Even at ordinary temperatures, when dealing with substances whose molal specific heats have not reached the Dulong and Petit limit of $3R$, one must conclude that the classical equipartition laws fail so far as the equilibrium between a solid and its vapor is concerned. The velocity distribution of the vaporized atoms of carbon, say, is not that of the surface layers of the solid carbon from which the evaporation has taken place. Though the solid and its vapor may be in temperature equilibrium, the average translational kinetic energy of the vapor atoms need not, and generally will not, be the same as the average translational kinetic energy of the atoms of the solid. (That some prefer to ascribe the internal energy of solids to stationary waves does not alter the situation.) Even in the cases of mixtures of such simple gases as helium and argon, when the temperature is so low that the energy is confined mainly to small quantum numbers, the average kinetic energies of the helium atoms will differ from the average for the argon atoms. On the concept that the temperature is a measure of the average kinetic energy of the molecules, the argon and the helium, though in equilibrium, would have different temperatures. Quantum considerations very strongly oppose the concept of a translational energy-temperature proportionality.

Our precise definition of a temperature scale is based on an application of the Carnot cycle proposed by Lord Kelvin. Where Q_0 and Q_1 represent, for a particular cycle, the quantities of heat absorbed and liberated, respectively, by the working substance used in the Carnot engine, this scale requires the corresponding temperatures T_0 and T_1 to be so related that

$$T_0/T_1 = Q_0/Q_1. \qquad (2)$$

This scale is based on the assumption that energy is conserved in macroscopic processes, on the supposition that all possible p,v,T changes may be produced reversibly, and on the second law of

thermodynamics. Developed in the days of classical mechanics, these basic assumptions and the temperature scale are equally applicable to the phenomena of the present-day quantum mechanics.

Direct experimental application of the Carnot equation in measuring temperature is impractical. With its aid, however, other usable relations have been developed; for example, that for the Joule-Kelvin effect,

$$\mu = 1/C_p[T(dv/dt)_p - v] \tag{3}$$

and the Boltzmann equation for blackbody radiation,

$$\mathcal{R} = \sigma T^4. \tag{4}$$

Using chiefly Eq. (3), a precise temperature scale, ordinarily referred to as the *gas scale*, has been derived. It ranges from the gold point, 1336° K, down to about 1° K. There is great difficulty of application, however, near this lower limit. Above the gold point the radiation laws serve to carry the scale indefinitely upward.

Below the 1° K-limit of the gas scale, workers make use of well-founded thermodynamic laws and of the approximate Curie law for paramagnetic substances, namely,

$$I = CH/T, \tag{5}$$

where I represents intensity of magnetization, H field strength, and C a constant. The results are probably trustworthy. It would be interesting, however, to determine experimentally whether or not the net transfer of heat between neighboring bodies in this temperature region would be in accord with those measured temperatures.

What is important here is that neither the gas scale nor the "low temperature magnetic scale" seems to include any quantity expressible in strictly mechanical terms, which is strictly proportional to temperature.

The Boltzmann fourth-power, blackbody law is highly exact. However, it applies only to the radiation contained in, or originating within, an opaque-walled evacuated cavity of uniform temperature throughout, whose dimensions are large in comparison with the wave-lengths of the contained radiation. Although experimental application of the law is quite impossible at very low temperatures, we may consider what, in theory, it suggests regarding the temperature concept. It seems perfectly possible (1) to bring in contact with any body whatsoever whose temperature is desired another body shaped to form a cavity with opaque walls through which a small hole gives opportunity for observation of the blackbody interior, (2) to measure the radiancy $\mathcal{R}$ in watts per square centimeter, say, of the hole when the walls of the cavity have come into equilibrium with the body whose temperature is desired, and (3) to assign a temperature to the body in accord with the fourth-power law, that is, a temperature which would vary as the fourth root of the observed radiancy. For a case where the above-specified conditions are fulfilled, we should not hesitate, if convenient, to so assign a temperature, with the understanding that, except for experimental errors, such assignment could not be improved upon. Why not then identify the temperature of a body with the fourth root of the energy density within a contained, evacuated, opaque-walled enclosure? As in the case of the classically ideal gas considered earlier, there would be consistency. The natural cgs unit of temperature would then be the $(\text{erg/cm}^3)^{\frac{1}{4}}$, a temperature which we now rate at about 9350° K. This unit would be inconveniently large, but a small part of it might be chosen as a unit for practical purposes.

Were all bodies provided with extended opaque-walled evacuated cavities with small holes for observation purposes, there would be no inconsistency in identifying the temperature of a body with the fourth root of the energy density in its appropriate cavity. However, there are difficulties. First, though not all bodies have such blackbody cavities, each possesses a radiant energy density which only in the rare case is equal to the energy density of a blackbody cavity in equilibrium. Second, in view of the huge dimensions demanded for reduced temperatures, the *sine-qua-non* condition of an opaque-walled cavity cannot always be fulfilled. Third, liquids and gases cannot be shaped to give the desired cavities. Yet all solids and liquids and gases possess temperatures. It follows that the identification of temperature by means of the Boltzmann equation is not possible. Temperature is

not just another name for radiant energy density or for the fourth root of such density.

Still another possibility needs consideration. In statistical mechanics, there occurs a parameter θ that has characteristics similar to those of temperature and is, in fact, identified with kT, k being the Boltzmann atomic constant. Given the total translational kinetic energy, the number of molecules and the type of statistics (Einstein's) applicable for an ideal gas, one may compute a value for θ. But the function used in the computation is very far from being a simple algebraic function of the fundamental quantities of mechanics. In addition, it involves a measure of the disorder of the system. What the function is for real gases, liquids and solids is quite uncertain, if indeed a universal function exists.

At present we know of no purely mechanical quantity—that is, one expressible in terms of mass, length and time only—which can be used, however inconveniently, in place of temperature. We are inclined to conclude that temperature is itself a basic concept.

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A Normal Mks System of Units

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AMONG the many questions that arise in a study[1] of systems of electric and magnetic units, there are several which seem to the writer worthy of further and more detailed consideration. The most important of these is the role that the speed of propagation of electric effect plays in any system of units. The appearance of this factor early in the general expression for electric and magnetic forces clarifies the choices made in defining the various systems of units and adds physical meaning to the constants.

Another question which has not received a great deal of emphasis is the need to change the definition of magnetic moment to correspond with the trend away from poles toward currents. The permeability of the surrounding medium is not descriptive of the coil itself, yet it appears in the conventional expression for the magnetic moment of a coil. Rather closely allied with this is the convenient unit *ampere turn per meter* for H, which represents not so much the resultant force of one current on an element of another as it does the collection of currents together as intensity of source.

The advantages of the so-called rationalized form of the mks system of units have been stressed,[1, 2] while the study of its inherent weaknesses is often neglected. These questions are discussed and serve as an introduction to a *normal* form of mks units, which consists principally in expressing in mks units the values of the constants in the general equation for electric and magnetic forces. The normal form includes the *ampere turn* as the unit of H, which is one of the principal advantages of the rationalized form, and this form may be used either in place of, or parallel with, the rationalized form without ambiguity of symbols.

How much of the present controversy on units results from failure in the past to unscramble the dielectric constant and permeability cannot be stated; but the effect of this failure to be specific is presumably large. Much has been accomplished in recent years in separating[3] the pure ratios, dielectric constant and permeability, from other factors which depend on choices of systems of units. There are some advantages in unscrambling these factors still further, a gain not unlike that in teaching a child, before he learns to spell, to recognize and pronounce the entire word *rabbit* rather than a too abbreviated form "ra't" which calls for the explanation that the apostrophe represents letters (properties) of the alphabet. For example, the college freshman

[1] Am. J. Phys. (Am. Phys. Teacher) **6**, 144 (1938).
[2] Kennelly, J. Eng. Ed. **27**, 290 (1936); Am. Phil. Soc. **76**, 343 (1936).
[3] Webster, Am. J. Phys. (Am. Phys. Teacher) **2**, 149 (1934); Harnwell, *Electricity and electromagnetism* (McGraw-Hill, 1938).

can use the explicit form of electrostatic force in mks units, $f=c^2qq'/10^7r^2$, with the understanding that the factor 10^7 appears because of the shift of the unit of energy from the erg to the joule, and c^2 appears because the ampere is defined by magnetic forces which differ from the electrostatic forces of the same charges by the ratio vv'/c^2. The dimensionless dielectric constant κ is omitted in this specific expression for all charges, since in order to include κ, one must exclude from q the polarization charges of the medium. The usual form of the Coulomb law in rationalized mks units, $f=qq'/4\pi\epsilon r^2$, may be obtained conveniently by setting the permittivity[2] ϵ equal to $10^7\kappa/4\pi c^2$; and this appears more satisfactory than introducing ϵ in the Coulomb law together with the equivocal explanation that ϵ is a "property of the medium."

Dimensions

The operational viewpoint—that the meaning of physical quantities arises from the way they are measured[4]—indicates that the dimensions of a physical quantity representing the concept, should follow closely the operations made in determining that quantity. The arbitrary intelligent decision of the International Committee on Weights and Measures to base electrical quantities on absolute measurements of the mechanical forces of charges and currents, and the choice long ago of three primary mechanical units, require the use of *three* primary units in electricity if these units are to represent the measurements made. It is clear that, according to the operational viewpoint, the physical concept, as well as the measurements, may be represented the better by *three* primary units.

Another argument in favor of the three-dimensional systems is that the speed of light c appears naturally, while the four-dimensional systems have a greater tendency to obscure c in the factors ϵ and μ. That the speed of electric waves c properly appears in the units is evident from Maxwell's theory of electromagnetic waves; for Maxwell showed that electrostatics plus magnetism plus certain relations between these yield propagation of electric effect with speed c. Conversely, electrostatic forces plus propagation with speed c and the same interrelations yield magnetic forces. Dimensional analysis[5] likewise indicates the need of c. When currents are expressed in terms of charges moving with speed v, the ratio v/c of speed of charge to speed of electric effect appears explicitly. Thus it is legitimate to regard magnetic forces as the modification of electrostatic forces required by motion of charges, and even to consider gravitational forces as due to relative acceleration of charges. It is quite possible that reforms in electromagnetic theory will involve revisions in the expressions for magnetic forces as, for example, in Milne's equations of electromagnetism,[6] and the units the physicist must use should be readily adaptable to such changes.

The four-dimensional systems have some advantages in dimensional checking, and in representing important physical quantities. Yet in mechanics we get along very nicely by giving such fundamental quantities as force and energy derived and not primary units, and it is not always wise to go back to the primary units every time one wishes to check an equation dimensionally. In electricity four (or more) units, two of them derived and two of them primary, may be freely used. Since the ampere is the electric quantity most directly measured, these units may well include the joule, the ampere, the meter and the second.

The use of three primary units rather than four results in some electrical quantities having different dimensions in the theoretical system than in the mks system without recourse to laying the blame on any unknown natural dimensions of dielectric constant or permeability. These differing dimensions arise simply because the two systems have different definitions of electric charge, the one based on electrostatic force and the other based on magnetic force. Although the appearance of two sets of dimensions for a single physical quantity seems disconcerting at first, it serves to remind us that we have not yet reached the utopia where a single system of units with a single definition for each quantity is satisfactory for everyone. It

[4] Bridgman, *Logic of modern physics*. We may add as a corollary that the definiteness of one's physical picture of a quantity increases with the number of times he measures or computes it in the same way.

[5] Bridgman, *Dimensional analysis*, p. 13.

[6] Milne, Proc. Roy. Soc. **A165**, 326 (1938).

serves to remind us also that the dimensions of a physical quantity are a shorthand statement of relationships with other quantities and not ultimate natural dimensions. Many physical quantities such as speed, $v=l/t$, and area, $A=ab$, are always defined and measured in the same way,[4] and their dimensions appeal to us as being very natural.[7] If no mechanical action can take place without invoking electric forces of the electrons and protons of the bodies concerned, then apparently one can apply Kennard's criterion to the Coulomb law, increasing masses in the ratio μ, distances in the ratio λ, and accelerations in the ratio α so that charge is increased in the ratio $(\mu\alpha\lambda^2)^{\frac{1}{2}}$.

RATIONALIZATION

Giorgi[8] has stated that rationalization does not consist of putting the 4π of the formulas in its place, which is only a formal and supplementary advantage, but consists of admitting that the two constants of the ether—ϵ and μ—are quantities which involve a fourth physical dimension independent of the mechanical ones. In contrast to this statement is the widespread recognition of the *three*-dimensional Heaviside-Lorentz system also as rationalized. Introduction of the 4π in the denominator of the Coulomb laws so that it will disappear elsewhere is sometimes spoken of as *subrationalization*. If the rationalized mks system were adopted and taught exclusively to beginners, one can well imagine that in a few years' time instructors might become very tired of the question, "What is the use of (or what is there rational about) introducing 4π in the denominator only to be multiplied by the permittivity, $10^7/4\pi 9(10)^{16}$, so the 4π divides out?"

The principal advantage of subrationalization in the theoretical Heaviside-Lorentz system lies in field theory which describes propagation of light energy. Even here its advantage is small. The field equations expressed in terms of E and H are more convenient with the 4π removed and placed in the Coulomb laws, but then expressions for fields and potentials in terms of the charges which produce them are more complicated. If, in the future, field theory should become expressed more in detail as the statistical sum of retarded action and energy of one charge on another, this small advantage would presumably be lost. Furthermore, the omission of 4π in $D=E+P$ and $B=H+I$ invites misunderstanding when E and P, likewise H and I, are said to be quantities of different kinds. B is often assigned the particular meaning as the average of a microscopic H, but this meaning does not follow from the definition of B as the hybrid sum of H and I. It is desirable that quantities in the theoretical system, as well as in the more practical mks system, have a minimum of a variety of meanings. In this respect the four-dimensional systems would have the advantage of assigning different dimensions to P than to E, with $D/\epsilon_0=E+P/\epsilon_0$ as the equation in fields, and $D=\epsilon_0 E+P$ as a corresponding equation in charge density on condenser plates and dielectrics. It would not do, of course, to maintain that giving P and E the same dimensions requires them to be of entirely the same nature. Use of the form of the equations in Gaussian units, $D=E+4\pi P$ and $B=H+4\pi I$, can reduce the ambiguity, since one may utilize the simple appearance of the shape factor 4π as a reminder that $4\pi P$ is not polarization P itself but is a field due to charges displaced in the dielectric, similar to $E=4\pi Q/A$ for the field of surface charges. In brief, theorists can go just as far with the Gaussian system as with the Heaviside-Lorentz system, and the former is the more easily interpreted by the non-theorist. Fortunately, in the mks system, the various interpretations are more distinctly separated.

A disadvantage of rationalizing the mks system with the implicit assumption that it be accompanied by the subrationalized Heaviside-Lorentz system, rather than the Gaussian system, is that besides elimination of the familiar unit of flux density, the *gauss*, transformations from Heaviside-Lorentz units to mks units involve an irrational factor $(4\pi)^{\frac{1}{2}}$. This arises from the shift in size of the unit of charge in Heaviside-Lorentz units to $1/(4\pi)^{\frac{1}{2}}$ that of the electrostatic unit, while the coulomb is retained and the 4π in the denominator in the rationalized mks system is compensated by another 4π included in ϵ_0 and μ_0. For example, numerical transformations for charge, from Gaussian units to mks and from

[7] Kennard, Am. J. Phys. (Am. Phys. Teacher) **6**, 120 (1938).

[8] Giorgi, Elettrotecnica **19**, No. 13, May, 1932.

Heaviside-Lorentz to mks, are, respectively,

$$Q_{\text{coulombs}} = (1/10c)Q_s$$

and $$Q_{\text{coulombs}} = [1/10c(4\pi)^{\frac{1}{2}}]Q_{\text{Hlu}},$$

with $c=3(10)^8$ meters/sec.

It is recognized that the "rationalized" or "subrationalized" forms are no more rational than other "nonrationalized" forms. It would be unfortunate for students to learn by inference that all other forms of units are not reasonable. And from the viewpoints of operational theory and propagation theory Giorgi's specification of "rational" is not very reasonable. It may be well to drop the term *rational* altogether.

Permeability, Magnetic Field, Source Intensity and Source

In the electromagnetic system of units, permeability, represented by the symbol μ, could be considered either as a pure ratio of fields, $B/H=B/B_0$, or as a "property of magnetized media," B/H. The latter meaning has been carried over into the mks system[2, 3] to make $\mu_0=4\pi/10^7$. The dimensionless ratio of fields, sometimes expressed[9] as μ_r, is a distinct physical concept. If μ is retained for this pure ratio, which deserves a name and symbol of its own corresponding to κ in the electrostatic case,[10] then, to avoid differing meanings for the same symbol, a new symbol may be used to represent the product, $4\pi\mu/10^7$; η, which resembles μ upside down, may serve for the symbol[11] and is used in the remainder of this paper. If the name *inductivity*[9] comes into use for $4\pi\mu/10^7=\eta$ and $4\pi/10^7=\eta_0$, the descriptive term *permeability* can be retained for $\mu(=\mu_r=B/B_0=\eta/\eta_0)$, the value of permeability listed in tables.

The various meanings assigned to B and to H cause much confusion, and if physicists and engineers can compromise to the extent of using a number of unambiguous and single valued symbols, each group adopting from this larger number only those most convenient for its purposes, they will have accomplished something. The engineers have appropriated H as representing the cause of magnetization and have, in their characteristic realistic fashion, given it the unit, *ampere turn per meter*, of the physical cause, concentration of current. The physicist can afford to relinquish the symbol H to this meaning and, where he previously used H, use B_0 as equal to the force per unit current element due to another current, and he may use B' or B_0 as B/μ.

Assigning the next choice to the physicists, we find the term *field* applied to B in the sense that B is the average of a microscopic field. This seems very reasonable, as the term *field* qualitatively is a region in which some quantity experiences a force, and the force on a moving charge due to all sources is $\mathbf{f}=e\mathbf{v}\times\mathbf{B}$, an expression conforming to the trend away from fictitious poles toward currents.[12] But now a new name is needed for the engineers' H, that collection of currents together expressed in ampere turns per meter which serves as a cause of B. The descriptive term, *intensity of source*, is suggested.* In the solenoid, H is simply the concentration of ampere turns per meter. The equivalent source intensity at any point may be computed from $\mathbf{H}=\oint i d\mathbf{l}\times\mathbf{r}/4\pi r^3$. For the long straight wire it becomes $H=i/2\pi r$.

The line integral, $\oint Hdl$, about a closed path becomes the source itself in ampere turns, and an accordant name is *magnetomotive source*. In fact, the term *magnetomotive force* never was particularly appropriate for the quantity whose magnitude was work, not force, per pole.

Magnetic Moment, Intensity of Magnetization and the Fictitious Pole

The magnetic moment of a coil which is descriptive of the coil itself and does not include the permeability of the surrounding medium is[13] $M=NiA$ rather than ηNiA, and conforms to the definition of magnetic moment as maximum torque per field, $\mathbf{L}=\mathbf{M}\times\mathbf{B}$. The presence of μ or η in the numerator of the force equation

[9] A. A. P. T. Committee on Electric and Magnetic Units, Am. J. Phys. (Am. Phys. Teacher) **6**, 147 (1938).

[10] Birge, Am. J. Phys. (Am. Phys. Teacher) **2**, 41 (1934); Webster, reference 3.

[11] The symbols ν and γ, mentioned in reference 9, are not entirely suitable as they are often used for other related quantities, as in reference 2.

[12] Curtis, *Electrical measurements*, p. 5.

* Since this manuscript was presented, an article by A. Sommerfeld has appeared, Ann. d. Physik **36**, 336 (1939), in which he calls B field intensity and H magnetic excitation.

[13] Abraham, Bull. Nat. Res. Council, No. 93, pp. 27–29, Dec., 1933; reference 9, footnote 5.

appropriately describes the increased force on either of two coils immersed in a paramagnetic medium, the battery supplying the energy to orient the atoms of the paramagnetic fluid. It appears also as a proper description for two magnets immersed in the paramagnetic medium, for in this case the work done in orienting the atoms in the medium may be considered to come from the energy of the magnet whose alinement of atoms, amperian currents and magnetic moment should then decrease. It is this magnetic moment, capable of decreasing when inserted in the paramagnetic fluid, that is the vector sum of the magnetic moments of the atoms of the magnet, when these atomic magnetic moments, eA/τ, are expressed in terms of orbital motion or spin of charge e with period τ. The conventional magnetic moment $M_c=\eta NiA=(4\pi\mu/10^7)NiA=(4\pi\mu/10^7)\Sigma eA/\tau$ remains constant by definition when the magnet is immersed in the magnetic fluid.

The intensity of magnetization, $I=M/v$, becomes measured in amperes per meter. It seems fitting that intensity of magnetization I and source intensity H should bear the same partial name, *intensity*, and be expressed in the same unit. The distinction between them is clear, as H is the current per meter in a coil controlled by a battery or generator while I is the amperian current per meter produced in a rod of iron by the alinement of atoms.

If desired, the magnetic pole strength may be defined as $m=M/l$. The Coulomb law for poles then has a μ in the numerator, as is suggested by Sommerfeld,[14] $f=\mu mm'/10^7r^2$ newtons. The conventional pole, $m_c=M_c/l$, gives this law as $f=m_cm_c'/4\pi\eta r^2=10^7m_cm_c'/16\pi^2\mu r^2$ newtons.

The size of M_c is somewhat inconvenient. For a magnet 12 cm long, poles 10 cm apart, 0.5 cm² area and magnetic moment 2000 emu:

$M=2000$ emu,	$I_{Av}=333$ emu,
$M=2.00$ amp m²,	$I_{Av}=333(10)^3$ amp/m,
$M_c=8\pi(10)^{-7}$ newton m²/amp,	$(I_c)_{Av}=0.419$ weber/m²,

$m=200$ emu,
$m=20$ amp m,
$m_c=25.1\mu$ weber.

[14] Sommerfeld, *Pieter Zeeman, Verhandelingen* (Martinus Nijhoff, 1935), p. 161.

Building Systems of Units

The relation between electric and magnetic forces is made clearer when magnetic forces are expressed in terms of currents and moving charges, in line with measurements and with recent definitions of the ampere. The general expression for electric and magnetic force is

$$\mathbf{f}=aq_2\int \mathbf{r}\frac{dq_1}{r^3}+bi_2\mathbf{l}_2\times\oint\frac{i_1d\mathbf{l}_1\times\mathbf{r}}{r^3}, \qquad (1)$$

where a and b are constants. When the current is considered solely as charge q moving with velocity $\mathbf{v}$, $i=q/t$, $q\mathbf{v}=i\mathbf{l}$, and

$$\mathbf{f}=aq_2\int \mathbf{r}\frac{dq_1}{r^3}+bq_2\mathbf{v}_2\times\oint\frac{dq_1\mathbf{v}_1\times\mathbf{r}}{r^3}. \qquad (2)$$

Simple dimensional reasoning gives at once $b=a/c^2$, where c is a speed. Measurements of electric and magnetic forces give c the value of the speed of light. The various systems of units are chosen by assigning arbitrary values to the constants a and b.[15]

In the electrostatic system, electric charge q is defined by the electrostatic force by omitting a, formally making $a=1$ and dimensionless; then $b=1/c^2$ and Eq. (1) becomes

$$\mathbf{f}=q_2\int \mathbf{r}\frac{dq_1}{r^3}+i_2\mathbf{l}_2\times\frac{1}{c^2}\oint\frac{i_1d\mathbf{l}_1\times\mathbf{r}}{r^3}. \qquad (3)$$

In the electromagnetic system electric current i is defined by the magnetic force by omitting b, formally making $b=1$ and dimensionless; then $a=c^2$ and Eq. (1) becomes

$$\mathbf{f}=q_2\int \mathbf{r}\frac{c^2dq_1}{r^3}+i_2\mathbf{l}_2\times\oint\frac{i_1d\mathbf{l}_1\times\mathbf{r}}{r^3}. \qquad (4)$$

The Gaussian system is defined by making $a=1$ and dimensionless and carrying one of the factors c with each i, so that i/c in Gaussian units is equal to i in electromagnetic units,

$$\mathbf{f}=q_2\int \mathbf{r}\frac{dq_1}{r^3}+\frac{i_2\mathbf{l}_2}{c}\times\oint\frac{i_1d\mathbf{l}_1\times\mathbf{r}}{cr^3}, \qquad (5)$$

which, when dealing with moving charges, may

[15] Warburton, Am. J. Phys. (Am. Phys. Teacher) 4, 125 (1936).

TABLE I. *Definitions and names of electrical quantities.*

Quantity	Gaussian Units		Normal mks Units	
	Defining Equation	Name	Defining Equation	Name
Charge	$\mathbf{f}=q_2\int \mathbf{r}dq_1/r^3$	statcoulomb	$q=\int idt$	coulomb
Current	$i=q/t$	statampere	$\mathbf{f}=i_2\mathbf{l}_2\times\oint i_1 d\mathbf{l}_1\times\mathbf{r}/10^7r^3$	ampere
Potential (Electromotance)	$V=W/q$	statvolt	$V=W/q$	volt
Resistance	$R=V/i$	sec/cm	$R=V/i$	ohm
Electric field	$E=f/q$	statv/cm	$E=f/q$	v/meter
Capacitance	$C=q/V$	centimeter	$C=q/V$	farad
Dielectric constant	$\kappa=C/C_0$		$\kappa=C/C_0$	
Permittivity			$\epsilon=\kappa\epsilon_0=10^7\kappa/4\pi c^2$	farad/meter
Electric moment	$p=ql$	statcoul cm	$p=ql$	coul meter
Polarization	$P=p/v$	statcoul/cm^3	$P=p/v$	coul/meter^2
Ideal field	$E'=D=\kappa E$	statv/cm	$E'=\kappa E$	v/meter
Displacement			$D=\epsilon E$	coul/meter^2
Magnetic field, **B**	$\mathbf{f}=(i\mathbf{l}/c)\times\mathbf{B}$	gauss	$\mathbf{f}=i\mathbf{l}\times\mathbf{B}$	newton/amp meter (weber/meter^2)
Magnetic flux	$\phi=\int\mathbf{B}\cdot d\mathbf{A}$	maxwell	$\phi=\int\mathbf{B}\cdot d\mathbf{A}$	weber
Source intensity			$H=Ni/l=\oint\frac{id\mathbf{l}\times\mathbf{r}}{4\pi r^3}$	amp turn/meter
Permeability	$\mu=B/B_0=B/H$		$\mu=B/B_0$	
Inductivity			$\eta=B/H=4\pi\mu/10^7$	weber/meter amp turn
Conventional field in iron	$B'=H'=B/\mu$	gauss	$B'=B/\mu$	newton/amp meter
Conventional intensity in iron			$H'=B/\eta$	amp turn/meter
Magnetomotive source	$\text{mms}=\begin{matrix}\oint Hdl\\ \oint H'dl\end{matrix}$	gilbert	$\text{mms}=\begin{matrix}\oint Hdl\\ \oint H'dl\end{matrix}$	amp turn
Vector potential	$\mathbf{A}=\int id\mathbf{l}/cr$	erg/abamp cm	$\mathbf{A}=\int id\mathbf{l}/10^7r$	joule/amp meter
Reluctance	$R'=\text{mms}/\phi$	gilbert/maxwell	$R'=\text{mms}/\phi$	amp turn/weber
Inductance	$\mathfrak{L}=V/(di/dt)$	sec^2/cm	$\mathfrak{L}=V/(di/dt)$	henry
Magnetic moment	$M=L/B\sin\theta$	abamp cm^2	$M=L/B\sin\theta$	amp meter^2
Intensity of magnetization	$I=M/v$	abamp/cm	$I=M/v$	amp/meter
Pole	$m=M/l$	abamp cm	$m=M/l$	amp meter

conveniently be written

$$\mathbf{f}=q_2\int\mathbf{r}\frac{dq_1}{r^3}+\frac{q_2\mathbf{v}_2}{c}\times\oint\frac{dq_1\mathbf{v}_1\times\mathbf{r}}{cr^3}. \tag{6}$$

Another system which we call the *basic* system may be defined by following the example of Mason and Weaver[16] except for the 4π, and making $a=b=1$ and dimensionless, provided $il=qv/c$ and $i=q/ct$ rather than $i=q/t$; then Eq. (1) becomes

$$\mathbf{f}=q_2\int\mathbf{r}\frac{dq_1}{r^3}+i_2\mathbf{l}_2\times\oint\frac{i_1d\mathbf{l}_1\times\mathbf{r}}{r^3}, \tag{7}$$

and, on substituting $l=vt$ and $i=q/ct$, returns to Eq. (6). This basic system conforms more closely to present usage than does the Gaussian system,[17] its units of resistance and inductance are of convenient size,

$$R_b=(1/30)R_{\text{ohm}},\quad \mathfrak{L}_b=(1/30)\mathfrak{L}_{\text{henry}},$$

and it has no more serious difficulties than giving potential and current the same dimensions, and introducing c in the definition of current and in the heating terms.

In the system of atomic units the charge on the electron is chosen as unity, the unit of distance is the radius of the 1*s* Bohr orbit of hydrogen, and the unit of energy is either that required to ionize hydrogen from its normal state or twice that amount. Choosing the latter, the electrostatic energy of two electrons is $1/r$ and $a=1$ and dimensionless, and all q's are replaced by the numbers z of electronic charges, Eq. (2) becoming

$$\mathbf{f}=\mathbf{r}\frac{z_2z_1}{r^3}+\frac{z_2\mathbf{v}_2}{c}\times\sum\frac{z_1\mathbf{v}_1\times\mathbf{r}}{cr^3}. \tag{8}$$

Transformation equations are $l=|4\pi^2me^2/h^2|l_{\text{cm}}=(1/0.53\times10^{-8})l_{\text{cm}}$, and $W=|h^2/4\pi^2me^4|W_{\text{erg}}=1/44\times10^{-12}W_{\text{erg}}$. The added equations connecting quantities in this system reduce the number of primary units to less than three.

The mks system of units is defined by retaining

[16] Mason and Weaver, *The electromagnetic field*, pp. 175, 233.

[17] Birge, Am. J. Phys. (Am. Phys. Teacher) **2**, 45 (1934). Combining in a single complete system present usages of parts of different systems is one of the goals sought.

TABLE II. *Formulas comparing normal and rationalized forms of mks units.*

NORMAL	RATIONALIZED	NORMAL	RATIONALIZED
Electrostatic force, f		*Magnetic force of currents, f*	
$(9\times10^9)qq'/\kappa r$	$(4\pi 9\times10^9/\kappa)qq'/4\pi r$	$il \sin\alpha \oint \mu i'dl' \sin\theta'/(10^7)r^2$	$il \sin\alpha \oint (4\pi\mu/10^7)i'dl' \sin\theta'/4\pi r^2$
Electrostatic field, E		*Magnetic field (Flux density), B*	
$\int (9\times10^9)dq'/\kappa r^2$	$\int (4\pi 9\times10^9/\kappa)dq'/4\pi r^2$	$\oint \mu i'dl' \sin\theta'/(10^7)r^2$	$\oint (4\pi\mu/10^7)i'dl' \sin\theta'/4\pi r^2$
Potential, V		*Source intensity in solenoid, H*	
$\int (9\times10^9)dq'/\kappa r$	$\int (4\pi 9\times10^9/\kappa)dq'/4\pi r$	Ni/l	Ni/l
Parallel plate condenser, C		*Field in solenoid and iron, B*	
$\kappa A/(9\times10^9)4\pi d$	$(\kappa/4\pi 9\times10^9)A/d$	$4\pi\mu Ni/(10^7)l$	$(4\pi\mu/10^7)Ni/l$
Cylindrical condenser, C		*Long straight wire*	
$\kappa l/(9\times10^9)2 \ln r/r'$	$(\kappa/4\pi 9\times10^9)2\pi l/\ln r/r'$	$H=Ni/2\pi r$ $B=2\mu Ni/(10^7)r$	$H=Ni/2\pi r$ $B=(4\pi\mu/10^7)Ni/2\pi r$
Capacitance of sphere, C		*Field at center of flat coil, B*	
$\kappa r/(9\times10^9)$	$(\kappa/4\pi 9\times10^9)4\pi r$	$2\pi\mu Ni/(10^7)r$	$(4\pi\mu/10^7)Ni/2r$
Wave speed, v		*Magnetic force of poles, f*	
$c/(\mu\kappa)^{\frac{1}{2}} = 3\times10^8/(\mu\kappa)^{\frac{1}{2}}$	$\dfrac{1}{(\epsilon\eta)^{\frac{1}{2}}} = \dfrac{1}{\left(\left(\frac{4\pi\mu}{10^7}\right)\left(\frac{\kappa}{4\pi 9\times10^9}\right)\right)^{\frac{1}{2}}}$	$\mu mm'/(10^7)r^2$	$(10^7/4\pi\mu)m_c m_c'/4\pi r^2$

the ampere, the meter and the joule. The ampere has been defined by the relation, $i_{\text{amp}}=10i_{\text{emu}}$, so that the volt is about equal to the emf. of a Daniell cell or a Clark cell; and $f_{\text{newton}}=W_{\text{joule}}/l_{\text{meter}}=f_{\text{dyne}}/10^5$. Substituting these values in Eq. (4) and comparing with Eq. (1), one finds $b=1/10^7$ and $a=c^2/10^7$ whether rationalized or not, and whether one considers he is using a four-dimensional or a three-dimensional system. The simplest choice is to make b dimensionless. Then

$$\mathbf{f}=q_2\int \mathbf{r}\frac{c^2dq_1}{10^7r^3}+i_2\mathbf{l}_2\times\oint\frac{i_1d\mathbf{l}_1\times\mathbf{r}}{10^7r^3}, \tag{9}$$

with $\mathbf{f}$ in newtons, $q[=it]$ in coulombs, and r and l in meters. This equation, containing the explicit factors c^2 and $1/10^7$, represents the *normal* form of mks units.

Although b is considered dimensionless, its magnitude may be determined by substituting values of current, length and force in Eq. (9). Then $b=\mathbf{f}/(i_2\mathbf{l}_2\times\oint i_1d\mathbf{l}_1\times\mathbf{r}/r^3)=10^{-7}$ newton/amp^2, just as a dimensionless angle may be determined by substituting values of force, length and work in $W=\int Ld\phi$, giving $\phi=W/L$ joule/newton meter. The derived unit of b, *newton per square ampere*, is the same derived unit obtained for b when the ampere is considered one of the four fundamental quantities of an mksa system (or when permeability or any other quantity is chosen as the fourth fundamental unit), whether the mksa (or mksμ) system be strictly four-dimensional using the international value of the ampere in vogue before 1940, or whether it be four-dimensional in concept only, with the absolute ampere determined by measurements of length, mass and time.

Likewise the value of a may be determined from

$$a=fr^2/q_2q_1=(3\times10^8)^2/10^7 \text{ newton meter}^2/\text{coulomb}^2,$$

whether one is using the three-dimensional mks system with the coulomb as a derived unit, or a four-dimensional concept with a three-dimensional measurement of the ampere and the coulomb, or a strictly four-dimensional system with the coulomb defined by the amount of silver deposited.

To reiterate, a may be expressed in *newton meter²/coulomb²* and b in *newton/ampere²*, while at the same time c is measured in *meter/second* and b is dimensionless.

The Heaviside-Lorentz system is defined by making $a=1/4\pi$ and dimensionless and carrying one of the factors c with each i, as in the Gaussian system. Then $b=1/4\pi c^2$ and Eq. (1) becomes

$$\mathbf{f}=q_2\int \mathbf{r}\frac{dq_1}{4\pi r^3}+\frac{i_2\mathbf{l}_2}{c}\times\oint\frac{i_1d\mathbf{l}_1\times\mathbf{r}}{4\pi cr^3}. \tag{10}$$

In the rationalized or subrationalized form of mks system, however, b is still 10^{-7} newton/amp^2, $a=(3\times10^8)^2/10^7$ newton meter^2/coulomb^2, as previously mentioned, and new quantities, ϵ_0 and η_0, which may be defined by $\epsilon_0=1/4\pi a$ and $\eta_0=4\pi b$, are introduced so that

$$\mathbf{f}=q_2\int \mathbf{r}\frac{dq_1}{4\pi\epsilon_0r^3}+i_2\mathbf{l}_2\times\oint\frac{\eta_0i_1d\mathbf{l}_1\times\mathbf{r}}{4\pi r^3}. \tag{11}$$

The foregoing discussion of the dimensions of a and b obviously applies to $1/\epsilon_0$ and to η_0, except for the numerical 4π.

If dq_1 and i_1 are restricted to exclude polarization charges and amperian currents, then (relative) dielectric constant κ and (relative) permeability μ should be included in Eqs. (1–11). In Eq. (11) ϵ may be written in place of $\kappa\epsilon_0$, and η for $\mu\eta_0$, when one desires not to unscramble.

Summary

Table I gives a series of definitions of electric quantities, for brevity, in equation form. Using κ and ϵ defined for the ideal case of uniform mediums of infinite extent, we have the ideal field $E'=\kappa E$ and the displacement $D=\epsilon E$ themselves defined in mediums of lesser extent where surface conditions must be taken into account. Similarly, with μ and η defined for mediums of infinite extent, conventional intensity H' is defined inside mediums of lesser extent by $H'=B/\eta$. A corresponding conventional field is defined as $B'=B/\mu$.

Table II compares the rationalized form with the normal form of mks units by expressing, in parentheses, the numerical values of η and ϵ or its reciprocal in the former, and in the latter the numerical values of the constants due to choice of units.

As typical illustrations of the normal and rationalized forms of the mks system and their relations to the theoretical systems, there are listed in Table III some common derived equations. Column 1 gives them in Gaussian units; column 2 gives the general form of these equations in any system of units including the mks

Table III. *Common derived equations.*

	mks Units			
	Normal			
Gaussian Units	General	Specific	Rationalized	Heaviside-Lorentz Units
$E=\int dq_1/\kappa r^2$	$=a\int dq_1/\kappa r^2$	$=\int c^2dq_1/10^7\kappa r^2$	$=\int dq_1/4\pi\epsilon r^2$	$=\int dq_1/4\pi\kappa r^2$
$V=\int dq_1/\kappa r$	$=a\int dq_1/\kappa r$	$=\int c^2dq_1/10^7\kappa r$	$=\int dq_1/4\pi\epsilon r$	$=\int dq_1/4\pi\kappa r$
$D=\kappa E=E+4\pi P$	$E'=\kappa E=E+4\pi aP$	$=E+4\pi c^2P/10^7$ $D=10^7E/4\pi c^2+P$ $E=(4\pi c^2/10^7)(D-P)$	$=E+P/\epsilon_0$ $=\epsilon_0E+P$ $=(D-P)/\epsilon_0$	$D=E+P$
$C=\kappa A/4\pi d$	$=\kappa A/4\pi ad$	$=\kappa 10^7A/4\pi c^2d$	$=\epsilon A/d$	$=\kappa A/d$
$\mathbf{B}_0=\oint i_1d\mathbf{l}_1\times\mathbf{r}/cr^3$	$=\oint bi_1d\mathbf{l}_1\times\mathbf{r}/r^3$	$=\oint i_1d\mathbf{l}_1\times\mathbf{r}/10^7r^3$	$=\oint \eta_0i_1d\mathbf{l}_1\times\mathbf{r}/4\pi r^3$	$=\oint i_1d\mathbf{l}_1\times\mathbf{r}/4\pi cr^3$
$B=H+4\pi I$	$=B_0+4\pi bI$ $=B'+4\pi bI$ $=4\pi b(H+I)$	$=B_0+4\pi I/10^7$ $=B'+4\pi I/10^7$ $=(4\pi/10^7)(H+I)$	$=\eta_0H+I_c$ $=\eta_0H'+I_c$ $=\eta_0(H+I)$	$=H+I$
$H=4\pi Ni/cl$ $B=4\pi\mu Ni/cl$	$=Ni/l$ $=4\pi\mu bNi/l$	$=Ni/l$ $=4\pi\mu Ni/10^7l$	$=Ni/l$ $=\eta Ni/l$	$=Ni/cl$ $=\mu Ni/cl$
$W/v=(\kappa E^2/8\pi)$ $+(B^2/8\pi\mu)$	$=(\kappa E^2/8\pi a)+(B^2/8\pi b\mu)$	$=(10^7\kappa E^2/8\pi c^2)$ $+(10^7B^2/8\pi\mu)$	$=(\epsilon E^2/2)+(B^2/2\eta)$	$=(\kappa E^2/2)+(B^2/2\mu)$
$W/v=(ED/8\pi)$ $+(BH/8\pi)$	$=\frac{1}{2}ED+\frac{1}{2}BH$	$=\frac{1}{2}ED+\frac{1}{2}BH$	$=\frac{1}{2}ED+\frac{1}{2}BH$	$=\frac{1}{2}ED+\frac{1}{2}BH$
$\text{div}\,\mathbf{E}=4\pi\rho/\kappa$	$=4\pi a\rho/\kappa$	$=4\pi c^2\rho/10^7\kappa$	$=\rho/\epsilon$	$=\rho/\kappa$
$\text{div}\,\mathbf{B}=0$	$=0$	$=0$	$=0$	$=0$
$\text{curl}\,\mathbf{E}=-(1/c)(d\mathbf{B}/dt)$	$=-d\mathbf{B}/dt$	$=-d\mathbf{B}/dt$	$=-d\mathbf{B}/dt$	$=-(1/c)(d\mathbf{B}/dt)$
$\text{curl}\,\mathbf{B}/\mu=(4\pi\rho\mathbf{v}/c)$ $+(\kappa/c)(d\mathbf{E}/dt)$	$=4\pi b\rho\mathbf{v}+(\kappa/c^2)(d\mathbf{E}/dt)$	$=4\pi\rho\mathbf{v}/10^7$ $+(\kappa/c^2)(d\mathbf{E}/dt)$	$=\eta_0\rho\mathbf{v}+\eta_0\epsilon(d\mathbf{E}/dt)$	$=(\rho\mathbf{v}/c)+(\kappa/c)(d\mathbf{E}/dt)$
$\nabla^2\mathbf{E}=(\mu\kappa/c^2)(d^2\mathbf{E}/dt^2)$	$=(\mu\kappa/c^2)(d^2\mathbf{E}/dt^2)$	$=(\mu\kappa/c^2)(d^2\mathbf{E}/dt^2)$	$=\eta\epsilon(d^2\mathbf{E}/dt^2)$	$=(\mu\kappa/c^2)(d^2\mathbf{E}/dt^2)$
$v=c/(\mu\kappa)^{\frac{1}{2}}$	$=c/(\mu\kappa)^{\frac{1}{2}}$	$=c/(\mu\kappa)^{\frac{1}{2}}$	$=1/(\eta\epsilon)^{\frac{1}{2}}$	$=c/(\mu\kappa)^{\frac{1}{2}}$

$c=2.998\times10^8$ meter/sec; $\epsilon_0=1/4\pi a=10^7/4\pi c^2$; $\eta=4\pi b=4\pi/10^7$; $\epsilon=\kappa\epsilon_0$; $\eta=\mu\eta_0$.

system; column 3 gives their specific form in the mks system; column 4 gives the rationalized form; and column 5 gives these equations in Heaviside-Lorentz units. In this table κ and μ appear in their usual places, dq_1 not including polarization charges, and i_1 not including amperian currents. Curl $\mathbf{H}=\rho\mathbf{v}+(d\mathbf{D}/dt)$ and curl $\mathbf{E}=-\eta d\mathbf{H}/dt$ are not included here in mks units since $\mathbf{H}$ is defined as a source intensity, and its space and time rates of change at a distance from the source have less concrete physical meaning than the changes in its field $\mathbf{B}/\mu$, and it is easier to visualize the rate of change of electric field $d\mathbf{E}/dt$ at a point in space due to charge on a conductor than it is to visualize the rate of displacement $d\mathbf{D}/dt$ of charge at a point in space where there is no charge.

The normal and the rationalized forms of the same mks system given in the tables provide unambiguous symbols for the use of the "rationalist" and the "normalist" and the "three-dimensionalist" and the "four-dimensionalist" while working agreeably side by side. By use of the quantities ϵ and η in Table II and in Table III, column 4, the rationalized form advocated by many physicists is preserved practically intact (curl $\mathbf{H}$ and $d\mathbf{H}/dt$ may be used if desired). On the other hand, emphasis on physical details of the part played by polarization and magnetization as distinct from factors representing units, is given by the quantities a, b, κ, μ and c in Tables I and II and in Table III, columns 2 and 3. One may use, for example, column 2 or 3, Table III, without making the change from custom of placing the $4\pi\epsilon$ and $\eta/4\pi$ in the inverse square laws, and he may still have the convenience of $C=\epsilon A/d$ and $B=\eta Ni/l$ for the parallel plate condenser and the solenoid merely by substituting $\epsilon=\kappa/4\pi a$ and $\eta=4\pi\mu b$ only at the convenient places. No more new names are needed for the units in the normal form than in the rationalized form, and the "unrationalized" form[1,2] can be discarded.

One hesitates to suggest these new normal and basic systems in a field already cluttered with too many systems of units, yet they represent to a considerable extent the present status and usage of electric quantities. The aim has been to give precise statements of conditions and facts. The speed of light appears not as a "ratio between the units" but as a natural factor connecting electric and magnetic forces. The writer wishes to express appreciation to members of the erstwhile A.A.P.T. Committee on Electric and Magnetic Units and others for ideas included here.

A Student Interferometer

Andrew Longacre
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MOST textbooks rather glibly refer to the possibility of determining the wave-length of monochromatic light by measurements on the interference pattern obtained with a thin wedge of air. Because of the simplicity of the arrangement and the fact that all of the quantities required for the evaluation are, apparently, directly measurable, this experiment has always appealed to the author as desirable for a first course. However, in practice, while the experiment serves as a good demonstration,[1] it is quite difficult to render quantitative, as anyone who has tried knows. This is mainly because one must have either large, optically flat, plates of glass or else a piece of metal foil to separate the plates at one edge, which is too thin to measure precisely with a micrometer.

The apparatus shown in Fig. 1 utilizes the method and successfully overcomes the difficulties. The wedge of air is formed between two pieces of plate glass, A and B, one of which is small and is carried by a tilting beam. By changing the angle of the beam the angle of the wedge can be altered and the wave-length evaluated

[1] Sutton *et al.*, *Demonstration experiments in physics* (McGraw-Hill, 1938), p. 396.

from the changes in the separation of the bands of the pattern. Since successive black or bright bands appear every time the thickness of the wedge has increased by $\lambda/2$, it is apparent from Fig. 2 that

$$\lambda = \frac{2x}{D\Delta(1/d)},$$

where D is the length of the long leg of the tilting beam, x is the height of the far end of the beam and d is the distance between successive bands of the interference pattern.

The tilting beam has the form of a cross with one very long leg. This leg extends about 2 cm beyond the crosspiece so as to provide a tongue to which the glass plate A is cemented. The other end of the leg is bolted to the end of a 96-cm length of 1-in. brass tubing. The underside of the far end of this tubing is cut away for about 3 cm to a depth of 5 mm, and a small piece of glass, in which has been ground a small recess, is cemented to the exposed edges. When assembled, this recess rests on the point of the screw of the inverted spherometer which serves as a micrometer for measuring x.

The brass crosspiece is slit at each end for about 1.5 cm, and then, midway along each slit, is drilled and tapped to carry a steel machine screw. By squeezing the slits the screws are held quite tightly. The ends of the screws, which are sharpened, rest on the glass plate B, so that plate A is carried a half-millimeter or so above plate B. It is surprising how easily these screws can be adjusted so that the interference pattern is parallel to the arms of the cross.

Frequently two pieces of plate glass yield interference patterns when they are placed in

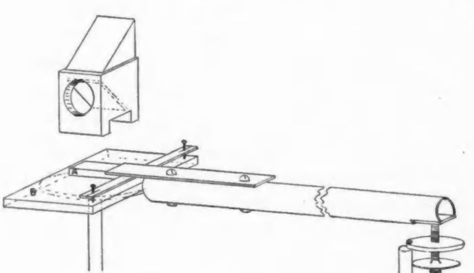


FIG. 1. Diagram of an adjustable wedge interferometer.

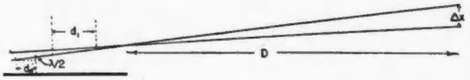


FIG. 2. Operation of adjustable wedge interferometer.

contact and illuminated by monochromatic light. For this work two pieces were selected which give a region of about 10 cm^2 where the bands are straight and nearly equally spaced when the plates are separated at one edge by a piece of very thin foil. The piece of plate glass B actually used was a refraction plate, 11×9 cm, while the piece A, measuring 2×5 cm, was cut from plate glass after the manner described by Reese.[2]

The thin wedge is illuminated by plane parallel, monochromatic light from a sodium flame or filtered mercury arc. The light is reflected onto the wedge by a piece of cover glass supported at 45° to the vertical in a small, wooden, bottomless box which straddles the plate A. The box also supports a 45° glass prism for viewing the pattern with a traveling telescope of short focal length.

The apparatus is assembled on a 1-m optical bench with the plate B resting on a prism table and the other end of the beam resting on an inverted spherometer which has been fastened to a post. The heights of the table and spherometer post are adjusted until an interference pattern is produced when the steel screws in the arm have been withdrawn so that they do not touch the plate B. Then these screws are advanced, alternatively a quarter of a turn or less at a time, until they lift the plate A off of B. Further adjustment of the screws will bring the pattern parallel to the crossarm. Plate A must be lifted high enough so that when the spherometer screw is advanced, the front edge of A will not touch plate B until the pattern has become quite fine.

The procedure in obtaining the data is first to lower the spherometer screw until the pattern is fine and to measure the width of 10 or more lines of the pattern at this setting. The spherometer screw is then advanced about 0.25 mm and the width of 10 or more lines is again measured. This is repeated until, after passing through parallelism, the pattern again becomes fine. It may be necessary to omit several sets of readings while plate A passes through the position where

[2] Reese, Am. J. Phys. (Am. Phys. Teacher) 4, 215 (1936).

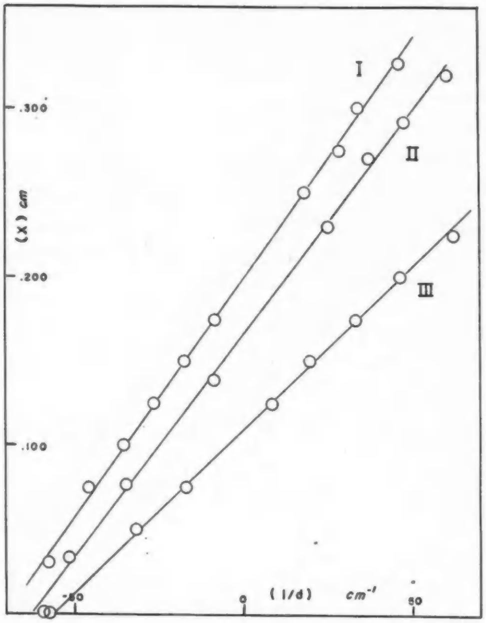


FIG. 3. Height of the long arm *vs.* reciprocal of the interference band width. Curve *I*, sodium D line and air wedge; curve *II*, mercury green line and air wedge; curve *III*, mercury green line and water wedge.

it is parallel to B because of the extreme width of the bands in this region.

The data for the graphs in Fig. 3 were obtained, with the help of a student, while using a plate A that showed marked curvature of the bands when it was nearly parallel to B. The ordinates are the readings of the spherometer in centimeters, and the abscissas are the reciprocals of the distance between centers of successive dark bands. The distance between 10 dark bands was measured with a small cathetometer reading only to tenths of a millimeter. In making the graph, the ordinates of curves I and II have been increased by 0.100 and 0.050 cm, respectively, in order to avoid having the curves cross near the zero abscissa. The abscissas were designated as + when the wedge of air was narrow on the left and − when the wedge was narrow on the right. The wave-length of light equals $2/D$ times the slope of the best straight line drawn through the plotted points. The data for curve *I* were obtained with light from a sodium flame, while those for curve *II* were taken with light from a mercury arc which had been filtered for the green line, 5461A. From curves *I* and *II* these wave-lengths were computed to be 5.85×10^{-5} cm and 5.50×10^{-5} cm, respectively, each of which is in error by only about 0.7 percent, though the probable error is undoubtedly greater.

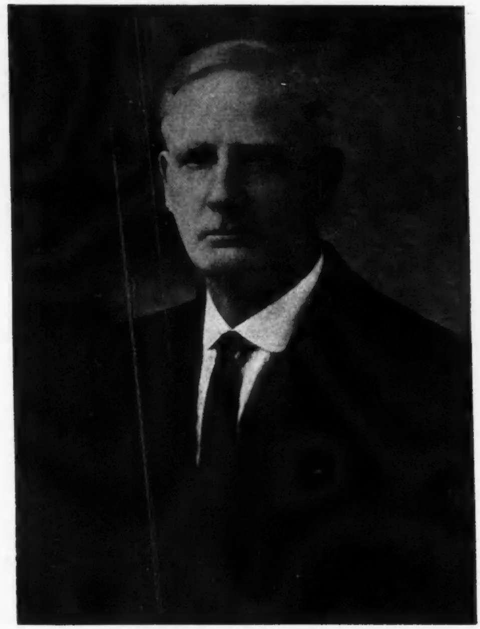
The data for curve *III* were obtained with the mercury green line after water had been introduced into the wedge by a medicine dropper. By accident some water got between plate B and the black top of the prism table. It was found that, while this decreased slightly the general intensity, it strongly increased the contrast of the pattern by reducing the light reflected from the lower surface of B. The pattern with the water wedge naturally had considerably less contrast than that with an air wedge and could be measured only in a darkened room. From curve *III* the wave-length of the mercury green line in water was found to be 4.10×10^{-5} cm. The ratio of the wave-length in air to that in water gives 1.34 as the refractive index of water.

The better students of the first course who have performed both this experiment and that of determining the wave-length with a transmission grating showed a marked preference for the former. They felt that they had evaluated every quantity used, whereas with the grating they had not. Furthermore, it appealed because it obviously used the phenomenon of interference to determine the wave-length of the very waves the existence of which it had revealed. They seemed to feel better about this than when interference is associated with diffraction. Finally, the measurements with the water carried, as they should, a complete conviction that the phenomenon of refraction is due to the relative speed of light in the two mediums.

I want to express my indebtedness to Dr. R. S. Bartlett for the suggestion of the water wedge, to those students who tried the apparatus in the various stages of its development, and to Mr. Everett Smith, who assisted in the construction of the apparatus.

Benjamin Harrison Brown

Recipient of the 1939 Oersted Medal for Notable Contributions to the Teaching of Physics



The American Association of Physics Teachers has made the fourth of its annual awards for notable contributions to the teaching of physics to Benjamin Harrison Brown, Emeritus Professor of Physics at Whitman College. The addresses of recommendation and presentation were made by Professor A. A. Knowlton, Vice President, and Professor Harvey B. Lemon, President of the Association, in a ceremony held on Wednesday afternoon, December 27, during the ninth annual meeting of the Association at Ohio State University.

Address by Professor A. A. Knowlton

THE work which any man does can be properly evaluated only when seen against the background of his environment. In introducing the Oersted Medalist of this year I have, therefore, thought it well to remind you briefly of the conditions under which he worked.

From the founding of Harvard University until the rather recent emergence of the tax supported institutions, higher education in the United States had a distinctly sectarian tinge. Everywhere along the westward moving frontier one found the church-supported missionary college. Usually without endowment or local support other than scanty tuitions, these institutions depended upon the generosity of people in longer settled regions to whom they were symbols of an idealism rarely paralleled in human activity. Under such conditions it was, of course, true that they offered little in the way of material inducements to prospective faculty members. Salaries were meager and sometimes uncertain, while advancement either within the institution or by transfer was not to be counted upon. On the other hand the requirements were severe in the extreme and were inherent in the nature of the institutions.

These colleges were founded upon an ideal and financed by men and women whose faith in that ideal reached all the way down to the pocketbook. In conformity with this ideal they were conceived

Two views of the Oersted collection in Copenhagen, Denmark, in which a copy of the Oersted Medal of the American Association of Physics Teachers is deposited. The present exhibit is temporary, as a building is under construction in which the collection will be permanently housed.

as institutions where young men and women were to be trained in character and attitudes, rather than as institutions of learning, in the narrower sense. If these ends were to be attained the missionary college must be staffed by men and women to whom these ideals were equally vital. New members of the faculty must be selected upon a basis of personal rather than scholarly qualifications. Faith in the ideal of higher education as an instrument of social salvation, eagerness to have a part in the missionary enterprise and a flair for life upon the frontier were essential. Occasionally, and far more frequently than one might have expected, the president of a missionary college was fortunate enough to secure the services of an individual in whom the requisite personal qualities were united with a rare capacity for continued growth in scholarship under the adverse conditions of isolation and long hours of teaching. Today it is my privilege to present to you as the Oersted Medalist of 1939 such a man.

Benjamin H. Brown,[1] Emeritus Professor of Physics at Whitman College, Walla Walla, Washington, was born in Wisconsin in 1866, graduated from Ripon Gollege in 1894 and received the Master of Arts Degree from that institution in 1896. He joined the faculty of Whitman College in 1895 and for some years taught physics, chemistry and biology. As the addition of new members to the faculty made it possible, he exchanged this general science bench for the chair of physics but never lost the breadth of scientific interest indicated by his earlier activities. Indeed, following his retirement in 1926 after thirty years of service, Professor Brown continued for some years to act as lecturer in geology and did much field work in this subject. His final contribution was the establishment of the Brown Foundation for the promotion of interest in astronomy.

During the entire period of his connection with Whitman College he was recognized by both faculty and students as an exceptional teacher. As the years passed students of physics from Whitman College appeared at the centers of graduate study in such numbers and of such quality as to make it clear that here physics was unusually well taught. I shall quote one of these men as to the characteristics of this teaching:

"First: he is at heart a dramatic character. To him nature is literally full of wonders, as revealed by man's progressive unfolding of her secrets. He is able to instill in the students a similar feeling of drama, to thrill them as he himself is thrilled. It is his dramatic instinct which serves as a key to excite and hold the interest of the students. Whether they are scientifically inclined or not, I do not recall ever seeing a drowsy student in his audience.

"Second: he is able to show how our physical knowledge is obtained without going into details which to the beginner may be tiresome. The essential experiments and deductions or inductions which lead to the physicist's viewpoint in any particular instance are often presented by him through a few simple arguments sufficient

[1] See also, S. B. L. Penrose, "Benj. H. Brown—A word portrait of a teacher of physics," Am. J. Phys. (Am. Phys. Teacher) 5, 161–166 (1937).

for the beginner, without mentioning details which for a more rigorous, logical development would be quite essential.

"Third: he has the faculty of stimulating individual thought. He can shock the student by a sweeping statement which may or may not be strictly correct, but which involves new lines of thought both interesting and worthwhile.

"Fourth: I admired his handling of figures and equations, the speed and dexterity with which he reached his goal. While this may have been due to long practice he never seemed to work from memory, and I have never had another teacher who excited a similar respect. I believe his skill lay in his presentation rather than in his ability as a mathematician, and I think it was particularly effective in the elementary courses.

"Fifth: As to his personality, he has a thrilling voice, an unusually earnest, modest, and friendly manner, and a striking figure. I have never heard of a student who disliked him, and his advanced students loved him unanimously."

Presentation of Award by Professor Lemon

SINCE Professor Brown, the medalist for 1939, is unable to be present because of his somewhat advanced years and the rigors of a long journey in the winter season, he is represented at this ceremony by Doctors Virgil Argo, Walter Brattain, Walker Bleakney and Vladimir Rojansky, members of a most distinguished body comprising his students. It is in such a group of younger men and women to whom the torch has been transmitted that a teacher finds his greatest satisfaction and through which his youth is eternally renewed. This group has selected one of their number, Doctor Argo, to represent Professor Brown. On behalf of the American Association of Physics Teachers I have the honor and pleasure of handing to Doctor Argo the Oersted Medal and the certificate of its award to Professor Benjamin Harrison Brown.

Apparatus for Demonstrations in Geometrical Optics

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Department of Physics, Brooklyn College, Brooklyn, New York

THE ability to demonstrate directly the fundamental phenomena of geometrical optics to large groups of students greatly facilitates the teaching of the subject. We have always felt the need for apparatus other than the usual optical disk for this purpose, since it is limited to use with small groups, and is rather critical of adjustment. The apparatus to be described embodies the well-known idea of making beams visible by scattering from smoke particles[1] and is large enough to be used in demonstrations before groups of approximately 100 persons.

The smoke box (Fig. 1) is a wooden box having the front and one end made of glass. The back is hinged at the bottom. A narrow glass observation window in the top enables the lecturer to make necessary adjustments from the rear. Bushings which support mounts for the optical accessories are inserted in a horizontal row in the

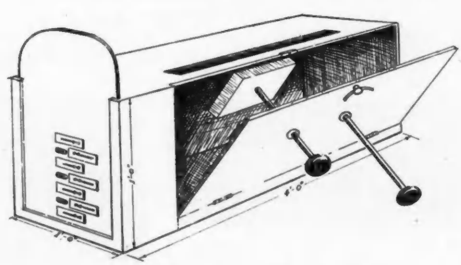

Fig. 1. The smoke box.

[1] Smoke boxes have been described by Clarke, *The science masters' book* (John Murray, 1931), part I, p. 65; J. Sci. Inst. **4**, Jan., 1927; Sutton *et al.*, *Demonstration experiments in physics* (McGraw-Hill, 1938), p. 372; Frick, *Physikalische technik*, vol. II, part 2, pp. 1152–53.

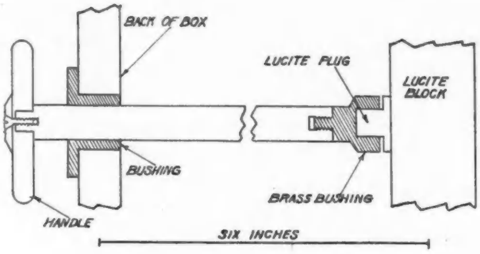

FIG. 2. Mounting for optical accessories.

back of the box and are located centrally in a vertical direction. The shutter is made of Bakelite, $\frac{3}{16}$ in. thick, and has seven horizontal slots each with its own shutter. The central slot is in the same horizontal plane as the bushings in the back, and the others are located symmetrically above and below the center. The entire shutter is removable. The accessories are mounted on $\frac{5}{8}$-in. brass rods that fit the bushings in the back (Fig. 2), are free to rotate and can be drawn back out of the beam. The entire box is painted dull black, both inside and out.

The illuminating system consists of a carbon arc, using 6-mm carbons, set at the principal focus of a simple lens of diameter 9 in. and focal length 14 in. (Bausch and Lomb No. 51–71–14). A lighted cigarette in an ash tray in the box affords a supply of smoke sufficient to last for about 4 hr. The apparatus could be mounted on a rotating table for use in wide lecture rooms.

The accessories we use are:

1. A plane, rectangular glass mirror mounted with one long edge toward the front of the box. The mirror is cemented to a metal plate, which is soldered in turn to the support rod.
2. A cylindrical metal mirror polished on both concave and convex surfaces. This is mounted by means of a chordal segment of metal, which is screwed to the edge of the mirror and also to the end of the support rod.
3. A plano-convex spherical lens of diameter 6 in. and focal length 10 in. At present we are using half of a cracked condenser from a projection lantern; but we intend to make a plano-convex cylindrical lens of lucite.
4. A double concave cylindrical lens.
5. A glass hemidisk.
6. A square parallelepiped of lucite, 1.5 in. thick, 6 in. on a side.
7. A 45°, 90° prism of lucite, 1.5 in. thick, cut from a 6-in. square.

Accessories 6 and 7 are of special interest since lucite has several properties that make it particularly useful for this work. It is obtainable in large pieces, is comparatively inexpensive, is readily cut and polished, is transparent and yet scatters enough light to make both the beams of light inside the object and the outline of the object readily visible. The two lucite blocks were obtained from E. I. duPont de Nemours Company for $6.78. They were first milled to shape and then polished. The polishing was done on a wet-cloth lap with pumice stone, then rottenstone and lastly rouge used as abrasives. The lap was a piece of smooth cotton cloth stretched over a metal plate. To produce the best quality of surface justified by the internal inhomogeneity of the lucite requires little skill or time. Once the technic is mastered, a surface

FIG. 5.

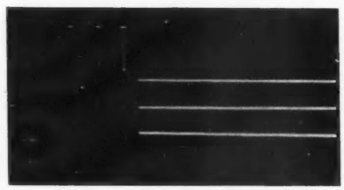

FIG. 6.

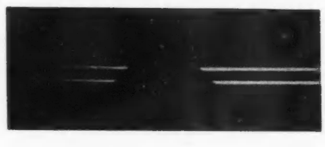

FIG. 3.

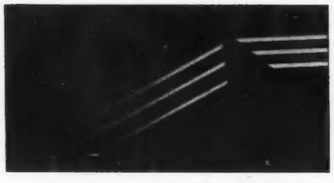

FIG. 4.

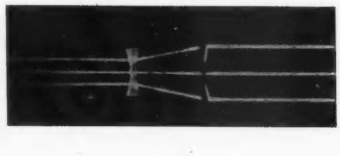

FIG. 7.

6×1.5 in. can be polished in about 30 min. The ease with which the lucite can be cemented also adds to its usefulness. Thus, in Fig. 2, a lucite plug is forced into a brass bushing and the lucite block is then permanently attached to the lucite plug by wetting both surfaces with chloroform and holding them in contact until dry.

With the apparatus as described we perform the following demonstrations:

A. Reflection at plane and curved surfaces (The caustic curve is readily visible).

B. Refraction phenomena in prisms.

1. Lateral displacement by a rectangular block (Fig. 3).
2. Deviation by a prism (Fig. 4).
3. Minimum deviation.
4. Dispersion by a prism.
5. Critical angle (The effect of wave-length is plainly visible).
6. Total internal reflection (Figs. 5 and 6).

It is interesting to note that in the refraction demonstrations the secondary beams resulting from reflections at all faces are quite visible.

C. Lenses.

1. Simple ray diagrams.
2. Spherical aberration.
3. Combinations of lenses (Fig. 7).

Figures 3–7, inclusive, are unretouched photographs of some of the demonstrations described. The outlines of the objects, though not apparent in the prints, are easily visible to the eye.

Appointment Service

REPRESENTATIVES of departments or of institutions having vacancies are urged to write to the Editor, Columbia University, for additional information concerning the physicists whose announcements appear here or in previous issues. *The existence of a vacancy will not be divulged to anyone without the permission of the institution concerned.*

Positions Wanted

27. Ph.D., physics, Northwestern '35; A.B., engineering, Harvard. Age 42, married, 3 children. Experience: 1 yr. lt., artillery; 12 yrs business and sales; 5 yrs college teaching. Interested in undergraduate teaching, including astronomy.

29. Ph.D., Northwestern; M.S., Pittsburgh; A.B., Muskingum. Age 34, married, 1 child. Has had 13 yrs teaching experience in two universities. Interested in teaching and research.

30. Ph.D., Univ. of Chicago. Many years experience as head of department of physics in prominent college. Author of books on physics and history of science. Large work on history of physics in preparation. Interested in college or university teaching.

31. Ph.D., Columbia. Years of experience as head of departments of physics in colleges and universities. Author of new type of laboratory manual. Designer of many new types of simplified apparatus. Research in radio, acoustics and methods of teaching physics.

33. M.S., experimental physics, coupled with thorough background of courses in professional education. Has taught physics and mathematics for 3 yrs in large high school. Desires position as instructor in high school physics in a university or college experimental or training school.

34. Ph.D., M.S., Penn State. Age 38, married. 13 yrs teaching experience in colleges and universities; 3 yrs head of department in small college; industrial research experience. Interested in teaching, research and administrative work in a small college.

35. Ph.D., Purdue; M.A., British Columbia. Age 27, married. Experience: 5 yrs university teaching; 2 yrs secondary school teaching; 5 yrs research in analysis of liquids by x-rays. Interested in teaching and research.

36. Ph.D., Pennsylvania '37; M.S., A.B., West Virginia. Age 30, married. Has taught 4 yrs in small liberal arts college of good standing. Interested in a position of greater responsibility and opportunity.

Departments having vacancies or industrial concerns needing the services of a physicist are invited to publish announcements of their wants; there is no charge for this service.

Any member of the American Association of Physics Teachers may register for this Appointment Service and have a "Position Wanted" announcement published without charge.

Graduate Appointments Available

Armour Institute of Technology, Dean L. E. Grinter, 3300 Federal St., Chicago, Ill. Apply before Feb. 20. Various academic and industrial scholarships, fellowships and assistantships, with stipends of $300 to $1200 less $150–300 tuition.

For specific information concerning types of graduate appointments and facilities in 96 other institutions, see the December, 1938, and February, 1939, issues of this journal, pages 342 and 72, respectively.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

9. The Earliest Print Showing a Steam Locomotive and Train

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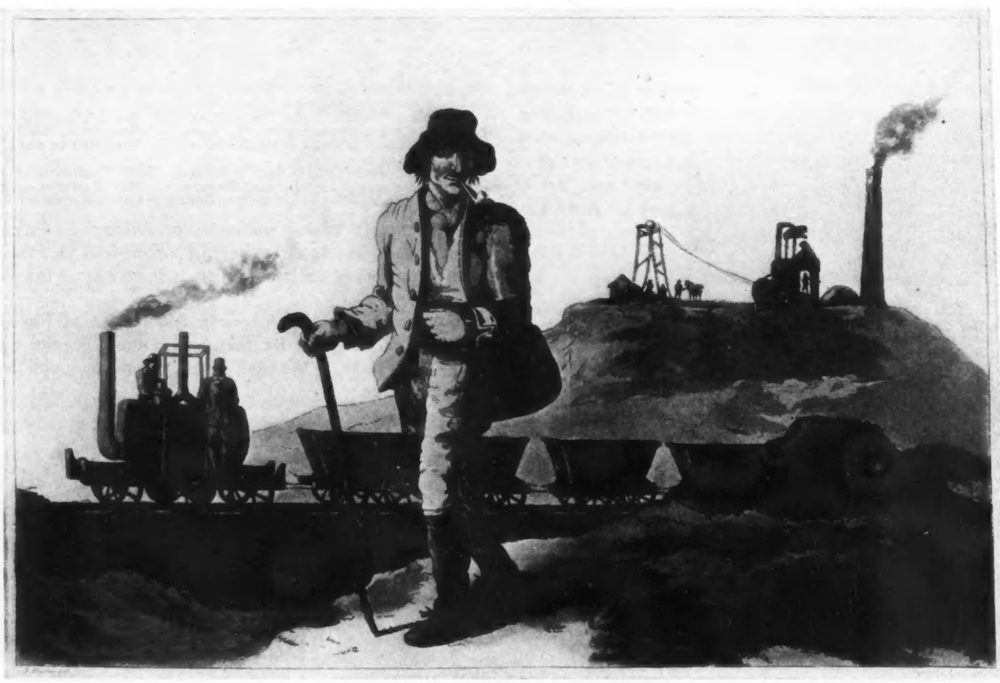
THE original of this print is an aquatint, 8×12 in., that was first published in 1813 and so is the earliest pictorial representation of a steam locomotive and train that we have; moreover the locomotive and train shown was the first to be commercially successful. The print first appeared in George Walker's *The Costume of Yorkshire* and was designed to show, not the steam train, but what the well-dressed miner of Leeds was wearing. It formed Plate III of the series and was entitled "The Collier." The description reads as follows:

> One of these workmen is here represented as returning from his labours in his usual costume. This dress, which is of white cloth bound with red, may probably be ridiculed as quite inconsistent with his sable occupation; but when the necessity of frequent washing is considered, surely none could have been adopted more conducive to cleanliness and health. The West Riding of Yorkshire, it is well known, abounds in coal, the consumption of which is prodigiously increased by the general use of steam engines. In the back ground of the annexed Plate is a delineation of the steam engine lately invented by Mr. Blenkinsop, agent at the colliery of Charles Brandling, esquire, near Leeds, which conveys about twenty waggons loaded with coals from the pits to Leeds. By two of these machines constantly employed the labour of at least fourteen horses is saved.

Richard Trevithick (1771–1833), whose high pressure noncondensing steam engine was the successful rival of Watt's low pressure steam



Blenkinsop's rack railway and the first commercially successful steam locomotive (from an aquatint by R. & D. Havell after G. Walker).

vacuum engine, was undoubtedly the originator of the steam locomotive. He constructed high pressure steam locomotives in 1801, 1803, 1805 and 1808. These engines were not commercially successful, however, as they were too heavy for the tracks on which they were used and too light to provide the traction necessary to haul loads great enough to enable them to compete economically with horses. To meet this difficulty JOHN BLENKINSOP (1783–1831), in 1811, patented a rack railway with teeth, cast on one of the rails, which engaged with a cogged driving wheel added to the engine. Toothed rails of this kind were laid in 1812 from the Middleton Colliery to Leeds, a distance of 3.5 mi, and four engines were built by Messrs. Fenton, Murray and Wood in 1812–1813 for use on them. The engines were based on TREVITHICK's designs, but embodied certain improvements due to MATHEW MURRAY (1765–1826). These were the first commercially successful steam locomotives and they remained in use for twenty years.

The Resolving Power of a Prism

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NEARLY every textbook of optics gives a proof of Rayleigh's expression for the resolving power of a prism,

$$R=\lambda/\Delta\lambda=Bd\mu/d\lambda. \quad (1)$$

Here $\Delta\lambda$ is the least difference in wave-length (in air or vacuum) between two bright lines that can be seen as separate, μ is the refractive index, which is different for different wave-lengths, and B is the width of the prism base, the dimension parallel to the transmitted beam. It is assumed that the prism is used at minimum deviation and that the beam is limited transversely by the prism. Any proof of Eq. (1) is based on the somewhat arbitrary criterion proposed by Rayleigh; in effect, that two bright lines, of wave-lengths λ and λ', where $\lambda'=\lambda+\Delta\lambda$, may be resolved if, in the Fraunhofer diffraction patterns in the focal plane of the telescope, the center or maximum of one pattern, due to λ', falls on the first minimum of the other, due to λ. Here it is the beam of wave-length λ that has the position of minimum deviation. It is supposed that the slit of the collimator is very narrow compared to any individual band in either diffraction pattern. In spectrometers designed and used for infra-red radiation the slits cannot ordinarily be so small, and the limit of resolution is determined mainly by the necessary slit widths, although affected somewhat adversely by diffraction.

The proof that $R=Bd\mu/d\lambda$, as given in some books,[1] involves the angular separation of the two beams and makes use of the angular dispersion of the prism, $d\theta/d\lambda$. R. W. Wood[2] quotes Rayleigh at length in a derivation that does not employ angular dispersion but does use and evaluate the angular separation of the two beams. It may be worth while to give here a very simple proof of the value of R. When one notices that the angular dispersion does not come into the final expression $Bd\mu/d\lambda$, he naturally suspects that angular separation and angular dispersion may be left out of the proof. The following argument owes its simplicity partly to this omission. In this respect the treatments by C. F. Meyer[3] and by G. Bruhat[4] are like the one to be given here, but they are longer, not so convincing, and not any more precise.

If the beam of light of wave-length λ (No. 1) is at minimum deviation, and the first minimum, or dark band next the bright center of its diffraction pattern, coincides in the focal plane of the telescope with the central maximum in the

[1] Houstoun, *A treatise on light* (Longmans, Green, 1934), p. 180; Jenkins and White, *Fundamentals of physical optics* (McGraw-Hill, 1937), p. 120.
[2] Wood, *Physical optics* (Macmillan, 1923), p. 108.
[3] Meyer, *The diffraction of light, x-rays and material particles* (Univ. of Chicago Press, 1934), p. 202.
[4] G. Bruhat, *Cours d'optique* (Masson et Cie., Paris, 1935), p. 227.

pattern due to the beam of wave-length λ' (No. 2), then it is not strictly true that the first minimum of No. 2 coincides with the central maximum of No. 1. For, since the wave-lengths are different, the two diffraction patterns are on different scales. But the wave-lengths differ so little that these scales are nearly the same. We are within the meaning of Rayleigh's criterion in either statement. We shall make the latter statement and say that the first dark band due to λ' falls at the center (or on the central maximum) of the pattern due to λ; or, in other words, that no light of wave-length λ' passes through the prism precisely in the direction taken by that part of the light of wave-length λ which goes to form the center of diffraction pattern No. 1. Referring to Fig. 1, where the beam shown is No. 1, no light of wave-length λ' leaves the prism along AJ because, in that direction, No. 2 would be entirely extinguished by interference.

Let JK be a wave front of No. 1 after the two refractions, and let AP, the plane bisecting the refracting angle, be a wave front of this light in the glass. The distance AJ is c and the distance PK, or $B/2$, is b. Let the refractive indexes μ

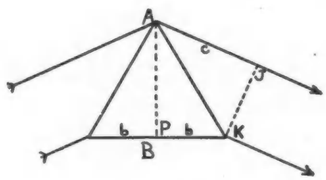


FIG. 1.

and μ' correspond to λ and λ', respectively. Then

$$b/(\lambda/\mu)=c/\lambda, \tag{2}$$

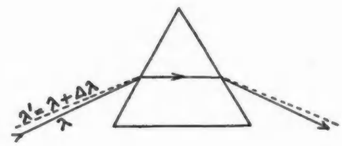
or

$$b\mu=c. \tag{3}$$

These equations express the fact that the number of wave-lengths from one wave front to another along one path is the same as the number along any other path. Also

$$b/(\lambda'/\mu')=(c/\lambda')+\tfrac{1}{2}, \tag{4}$$

FIG. 2.



or

$$b\mu'/\lambda'=(c/\lambda')+\tfrac{1}{2}. \tag{5}$$

This is because there is no light of wave-length λ' sent out in the direction AJ, and it must be that at J such light is one cycle different in phase from this same kind of light at K, and half that difference arises on each side of AP. The situation is like that when light is diffracted through a single slit. When a lens is so placed that parallel light from the slit is focused on the screen, the direction to the first dark band is determined by the fact that the two paths, from the two sides of the slit to the place of zero illumination, next to the bright center of the Fraunhofer pattern, differ by one wave-length.

Since $c=b\mu$, Eq. (5) becomes $b\mu'/\lambda'=b\mu/\lambda'+\frac{1}{2}$, or

$$b\Delta\mu=\lambda'/2. \tag{6}$$

Hence

$$B\Delta\mu/\Delta\lambda=\lambda/\Delta\lambda. \tag{7}$$

This follows because $B=2b$ and because it makes no difference whether we write λ or λ' as the wave-length concerned. This is Eq. (1), which was to be proved, if we use for $\Delta\mu/\Delta\lambda$ the derivative $d\mu/d\lambda$ at the wave-length concerned.

The broken lines in Fig. 2 indicate the direction taken by light of wave-length λ' when a beam of that light is at minimum deviation. If these lines are pulled down so as to fall along the full lines representing λ, no light would go through, for it is exactly in the direction taken by No. 1 that leads to the first dark band in the pattern due to No. 2.

By this same method also it is very easy to derive the expression $R=Nn$ for the resolving power of a grating.

Advance information of the result of the Battle of Waterloo consolidated the Rothschild fortune. It is advance information, which may be of equal potential value, that research offers you.—ARTHUR D. LITTLE.

Physics Instruction for Purposes of General Education[1]

A. A. A. S. Committee on the Improvement of Science Instruction for Purposes of General Education

THIS report represents one phase of a study undertaken by a special committee of the American Association for the Advancement of Science on the improvement of science teaching.[2] The attention of this special committee is devoted mainly to the problem of improving science instruction in colleges and universities for purposes of general education. The committee has assumed the following two responsibilities as an initial attack on the problem:

(1) To make a study of the current instructional practices in those courses that are designed primarily for purposes of general education, that is, of those science courses for students who take but few courses in science and for whom such courses are terminal.

(2) To determine those experimental studies that seem to be the most important and urgent to be carried on with the aim of improving science instruction for the nonscientist.

The committee decided that an effort should be made to determine the points of view of science teachers with respect to certain issues involved in the problem and also to discover the present practices in those courses designed primarily for purposes of general education. The committee decided, as a first step, to obtain some of this information by means of questionaries, one for each science field.

The purpose of the questionary was twofold:

(1) To obtain the reactions of science teachers to some of the issues involved in the problem of science instruction for purposes of general education.

(2) To locate those science departments that have given considerable thought and have had considerable experience with courses designed for the nonscientist.

Those departments which seem to have had considerable experience with this type of course were then contacted further by correspondence and personal visitation. In this way more detailed information was obtained concerning course procedures and contents, methods of testing, etc. The results tabulated in this report have to do with only the first aspects of this preliminary study, namely, the questionary.

The questionary concerning physics for purposes of general education was sent to approximately 500 colleges and universities. To obtain the opinions of a representative cross section of physics teachers concerning the problem, the questionary was sent to the 500 colleges and universities irrespective of whether the institution was known in advance to be favorable or unfavorable to the issues involved.

By August 1, 1938, two months after the questionaries were sent out, 194 of the 500 had been returned. They were distributed among the various types of institutions as follows: 149 colleges and universities; 33 teachers colleges; 12 professional colleges. In general, the response was made by the head or chairman of the Department of Physics. About 10 percent were answered by someone other than the Head of the Department. Five were answered by a committee of teachers within the department.

The following four major themes or questions were incorporated in the questionary:

(1) What do physics teachers believe about some of the important issues in the problem of physics for the purposes of general education?

(2) What do physics teachers believe should be accomplished in a physics course for the nonscience student?

(3) What has been done by various physics departments in an attempt to meet this problem?

(4) What are some of the major problems to be solved if physics instruction for the nonscience student is to be improved?

In order that the questionary would be as short as possible and the response large, only those issues and problems were included which have been most frequently mentioned. Additional comments were invited, and many teachers expanded their answers to the questions with several pages of written comments.

The Questionary and the Results

In preparing the questionary specific questions were formulated to obtain information concern-

[1] A preliminary report of the special committee of the American Association for the Advancement of Science on the improvement of science teaching in colleges and universities. This committee is made up of representatives from the fields of physics, mathematics, chemistry, botany, zoology and the earth sciences (geology and geography)—L. W. Taylor, Oberlin College, *Chairman*.

[2] Science **87**, 454 (1938).

ing the more general questions or items stated above. The questionary and the response to it by 194 physics teachers are given in the following table:

A relatively small number of the students who enter college physics continue their study in more advanced courses. A much larger group take the course in order to meet certain requirements or solely for its contribution to their general education. The assumption has usually been made that essentially the same type of course meets the needs of these different groups of students. One of the questions of general concern which this committee believes should be studied may be phrased as follows: Does the teaching of physics through its content and method of instruction now adequately meet the needs of those students who do not continue the study of the subject?

The committee would appreciate your cooperation in answering the following questions related to the foregoing problem. The committee makes no pretense that this questionary is complete and you may wish to add, on the reverse side of these sheets, comments related to the questions or to add other questions which are not presented in the questionary.

Questions	Responses of 194 teachers		
A. *Do you consider that the conventional introductory college course in physics as represented by a majority of current textbooks and laboratory manuals:*	*Yes*	*No*	*Uncertain*
1. Is in general satisfactory for the nonspecializing student?	64	106	26
2. Is more appropriate for students who later specialize in physics than for those who do not?	130	42	16
3. Could be significantly improved for the nonspecializing student?	119	33	39
4. If modified for the nonspecializing student would be in danger of becoming superficial?	101	57	34
5. Should be replaced for the nonspecializing student by a physical science survey course?	43	104	47
6. Would be adequate for the nonspecializing student as well if more time than is normally available were alloted to the course?	52	79	40
Do you consider that:			
7. The emphasis placed on "pure research" as a basis for advancement, has retarded the development of a real concern about, and research upon, teaching problems related to general introductory courses in physics?	137	38	22

B. *What do you believe are the most significant contributions that a study of physics should make to those students who are not to specialize in physics?* (Use the number *1* if you believe the contribution to be very important; the number *2* if you believe the contribution to be of some importance; the number *3* if the contribution is one which you believe a physics course should not attempt to make.)

The course should:	*1*	*2*	*3*
1. Develop the ability to think critically	165	30	1
2. Show how the discoveries of science have contributed to the "world-view" characteristic of the present scientific era	93	89	10
3. Show that certain prejudices have retarded the application of scientific discoveries to problems of everyday living	37	90	58
4. Develop the ability to treat problems quantitatively by means of equations and formulas	52	91	55
5. Develop certain manipulative skills involved in laboratory technique	41	103	54
6. Develop certain hobbies representing the interests of students	26	90	75
7. Make students familiar with the facts, principles and concepts of physics	160	37	5

C. *If your department has made some change in the introductory course in the last five years, what directions has it taken?*

	Yes	*No*	*Uncertain*
1. Has the content and program of instruction been considerably modified within the framework of the old course?	71	76	5
2. Has the department replaced an old course with a survey course?	24	140	0
3. Has the department introduced new courses (such as survey courses) in addition to the regular introductory course?	77	86	1
4. Do you rely mainly upon a single textbook and laboratory manual in the new or revised course?	91	48	2
5. Are you using an outline or syllabus which you have prepared especially for the new course?	30	97	2
6. Do you attempt to cover most of the traditional content such as mechanics, heat, light, sound and modern physics in the revised course?	97	35	4
7. Do you expect more outside readings in a survey course than you do in the course designed primarily for further work in physics?	67	38	6
8. Is the work in the new or revised course carried on independently of other physical science courses, such as chemistry, astronomy or geology?	91	36	1
9. Has your department prepared a bibliography of reading materials related to the interests of those people who may not specialize in physics?	49	94	2

10. Has your department prepared a list of problems requiring investigations which can be carried on by the student outside of the classroom and laboratory?	14	133	2
11. Has your department prepared special tests or other means of evaluating student achievement of the distinctive aims of the new or revised course?	36	104	3

D. *What are some of the things that you would like to have developed in a further consideration of the problems stated above?* (Indicate by the number *1* those things that you believe to be very important; by the number *2* those that you believe have some importance; and by the number *3* those that you believe are not at all important for this problem.)

	1	*2*	*3*
1. The clarification of a point of view for teachers concerning the place of science (and especially physics) in general education at the college level	147	37	7
2. The preparation of a list of problems suitable for purposes of general education which require:			
(*a*) Investigations making use of library materials	93	71	16
(*b*) Investigation requiring experimentation outside the laboratory	57	78	46
(*c*) Investigation in the laboratory	82	79	25
3. The preparation of reading material designed:			
(*a*) To develop the ability to see the implications of the discoveries and inventions of science for everyday living	131	49	9
(*b*) To encourage thinking as to how to overcome those prejudices that have retarded the application of scientific discoveries for the improvement of everyday living	74	77	30
4. The preparation of a bibliography of readings designed for purposes of general education and suitable for use in physics courses	111	60	20
5. The development of methods for discovering the particular needs and interests of students and for selecting science content and teaching procedures to meet those needs and interests	122	51	14
6. The preparation of tests designed to measure the achievement of students with respect to certain aims generally not now specifically tested (for example, understanding and use of the "scientific method" and the ability to think clearly	107	66	19
7. The development of technics for interpreting and using test results for purpose of improving the achievement of students	91	73	21
8. The training of those people already engaged in the teaching of science to meet the problem of science in general education	107	57	19
9. The institution of graduate courses (history and philosophy of science), these courses to be taught by scientists	81	66	33

Analysis and Interpretations of Results

The response to Part **A** of the questionary seems to indicate that the majority of those physics teachers answering the questions believe the regular introductory course in physics is unsatisfactory for the student who will not specialize in science. The responses also seem to indicate that physics teachers, in general, feel that the solution of the problem should not take the direction of modifying the regular introductory course so that it would meet more adequately the needs of the "nonscience" student. This inference is based on the belief of the majority of teachers that such a procedure would result in superficiality in the regular introductory course.

The analysis of the responses to Question **A**-4 (concerning superficiality in a modified regular course) suggests several interesting interpretations. One hundred and fifty-nine of the 191 persons responding to Question **A**-4 also answered Question **C**-1 (modification in the regular courses). These 159 responses to the two questions are distributed in the following way:

Question **A**-*4*		*Question* **C**-*1*	
Do you consider that the conventional introductory college course in physics as represented by a majority of current textbooks and laboratory manuals, if modified for the nonspecializing student, would be in danger of becoming superficial?		Has the content and program of instruction in the regular introductory course of physics been considerably modified within the framework of the old course during the last five years?	
(1) Yes	84	Yes 26	No 58
(2) No	46	Yes 16	No 30
(3) Uncertain	29	Yes 20	No 9

This analysis seems to indicate that the persons who are uncertain concerning superficiality if the regular course should be modified for the nonspecializing student have attempted a modification of the regular course to a greater degree accordingly than have the persons who are more sure (Yes or No) about this question.

The results of Part **A** also indicate that the majority of physics teachers believe the solution

to this problem is not to be found in a physical science survey course. Two things should be pointed out, however, with respect to this question. First, of the 104 teachers answering "No" to Question **A**-5, 23 indicated that they had tried some modification of the regular course in physics, Question **C**-1; 23 indicated that they had introduced some new type of course, one example of which may be the "survey" course, Question **C**-3; 9 indicated that they had both tried some modification of the regular course and introduced a new course, and 49 indicated that neither a modification of the regular course in physics had taken place nor had any new course been introduced. Second, 53 persons who had indicated that survey courses were given in their departments answered "No" or "Uncertain" to the question of offering a physical science survey course for the nonscience student. Of this group of 53, however, 34 represent departments that offer revised courses within the field of physics and 17 represent departments that offer courses combining physics and one or more of the other sciences. These data seem to indicate that a part of this group of 54 "No's" and "Uncertain's" to the question of offering physical science survey courses, was responding to the term *physical science* in the phrase "physical science survey course" rather than to the word *survey*. Eight other comments also indicated rather clearly that an additional course should be given but that the word *survey* did not describe the type of course that was desired.

The purpose of Part **B** of the questionary was to learn the beliefs of physics teachers concerning the contributions or purposes of physics instruction for the student who is not specializing in sciences. The purposes or aims of this type of course set forth in **B**-1 and **B**-7 (the development of critical thinking and the acquisition of information) seem to be the most important according to those who answered the questionary. However, many qualifying comments were made with respect to the aim of having students acquire information (**B**-7). Most of these qualifying comments could be included by rephrasing **B**-7 to read: "The course should make students familiar with *some* of the more important and useful facts, principles and concepts of physics." Implicit in these written comments seemed to be the conviction that it is not necessary for this type of student to be familiar with all of the facts, principles and concepts of physics which are presented to a student in the regular introductory course to physics, but that some physical principles and concepts are more important for this type of student than are others. The majority of comments also seemed to carry the conviction that it is better to have the student master those principles and concepts important for him than to have him cover the whole field of physics in a more superficial manner.

Part **C** was designed to obtain evidence concerning present practices utilized in courses primarily for the nonspecializing student. Almost 50 percent of those who answered this part of the questionary indicated that a new course had been introduced; an equal number indicated that some changes in the content or method in the regular course have been made. The data of Part **C** also indicate that in almost half of the new or revised courses the subject matter of physics is but one part of the course, which may include one or more of the sciences, such as chemistry, physics, astronomy, geology, and in rare cases, botany and zoology. The almost identical distribution of responses to Questions **C**-6 (regarding content of the revised course) and **C**-8 (dealing with the scope of the newer revised courses) seems to indicate that in the new or revised courses in the field of physics the content is somewhat the same as the regular introductory course in physics.

One practice in the new or survey courses which seems to be rather common is that of requiring more outside readings than are required in the regular introductory course. It also appears that relatively little has been done to evaluate the outcomes of the new or survey courses.

In Part **D** an attempt was made to determine the opinions of physics teachers concerning the importance of certain projects which, if adequately developed, may increase the effectiveness of physics instruction for the student not specializing in physics. Project **D**-1, "the clarification of a point of view for teachers concerning the place of science (and especially physics) in general education at the college level," seemed to be the concern of most of the teachers. Their interest in this project seems to indicate that a general

uncertainty exists concerning what physics instruction should do for nonscience students. Most teachers with whom the committee has had contact in addition to the questionary believe the present courses in physics are not designed to meet the needs of such students. But neither do they believe the solution to this problem may be found simply by making the regular course easier. These teachers very often raise the following questions in connection with this problem: "Which of the principles and concepts in physics should the nonspecializing student understand?" "What are some of the elements of critical thinking implicit in physics instruction which would be most desirable for the nonspecializing student to master?" "Are there certain attitudes which physics teachers consider as desirable outcomes of a physics course for the nonspecializing student?" These questions are usually raised when one begins to push back to the basic purposes of a physics course for the nonspecializing student.

The committee, of course, is not inferring standardization of a course for the nonspecializing student at this point but is simply restating some of the questions that have been raised concerning what this type of course should do. To the committee, Project **D**-1 means a clarification of the possible purposes of a course for the nonspecializing student. Such a clarification should include suggestions on such questions as "What is meant by understanding a physics principle or concept," "What is meant by critical thinking in the field of physics," What are some of the criterions which would enable a physics teacher to select and justify the inclusion of certain physical principles and concepts in his program of instruction?"

Project **D**-5, "The development of methods for discovering the particular needs and interests of students and for selecting physics content and teaching procedures to meet those needs and interests," ranked third in importance according to the judgment of most teachers. Of course, many of the problems involved in this project are inextricably related to the problem of formulating a point of view concerning science in general education, although many other related problems involve experimental studies which should be carried on. For example, one assumption which represents a trend in physics instruction for the nonspecializing student is that the experiences of students are more effectively carried on by means of demonstrations and lectures rather than through individual experimentation and discussion. Many physics teachers have expressed a desire for experimental evidence concerning this assumption so that they will be able to carry on more effective instruction in their courses.

At least three of the projects, **D**-3(a), **D**-4 and **D**-6, represent assistance which many departments would welcome. Since these projects apparently involve interrelated problems, possibly a coordinating committee which would delegate responsibilities to subcommittees constitutes one way of getting answers to some of the problems.

Is the Standard Introductory Course Adequate for Purposes of General Education?

The responses of physics teachers to the questionary seem to fall into two general categories representing opposite points of view concerning whether the typical course in physics is adequate to meet the needs of the nonscience student. Each category, however, involves several rather distinct differences of opinion.

That the method and content of the regular introductory course in physics are satisfactory for the nonspecializing student (72 in this group).

(*a*) Forty-nine of the 194 persons answering indicated rather clearly that they believe the present regular introductory course in physics to be satisfactory for the nonspecializing student. Of this group 8 indicated that some modification of the regular course in their department had taken place during the last five years, 4 indicated that a new or revised course (possibly a survey course) had been introducted, 4 indicated that both a modification of the regular course and the introduction of a survey course had occurred, and 33 indicated that neither a modification of the regular course nor the introduction of a new course had taken place. This group seems to believe that the regular course as it now exists is satisfactory for the nonspecializing student.

(*b*) Twenty-three of the 194 persons answering indicated rather clearly the belief that the regular introductory course would be satisfactory for the nonspecializing student if more time were available for it.

That the method or content of the regular introductory course in physics is not satisfactory for the nonspecializing student (120 in this group).

(*a*) Forty-five of the 194 persons answering indicated the belief that the regular course does not meet the needs

of nonspecializing students but that any attempt to provide physics instruction for these students should not take the direction of modifying the regular introductory course. This group is not unanimous, however, in the opinion that the physical science survey course should be the direction.

(*b*) Forty-three of the 194 teachers indicated that the solution to the problem of physics instruction for the nonspecializing student should take the direction of a new course, possibly a survey course.

(*c*) Thirty-two of the 194 teachers indicated that a modification of the regular course in physics would not result in superficiality in the regular course. But this group either rejected or were uncertain about the desirability of attempting to meet the problem through the introduction of a survey course. It might be inferred that this group would favor a modification of the regular existing courses in physics to meet the needs of those students who are not specializing in physics.

The committee has attempted to discover the problems that seem to be the most perplexing and stand in the way of progress in improving instruction for purposes of general education. The data obtained through the questionary and visits to colleges by two research assistants have served as basic material from which the committee will eventually make its recommendations.

A Study of Secondary School Physics

M. H. Trytten
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As a result of discussions on the teaching of physics in high school at various meetings of the Association of Physics Teachers of Western Pennsylvania and Environs, a committee was appointed to make a study of physics in the secondary schools. Two reports from this committee failed to offer any very brilliant suggestions, and its recommendations were not accepted. The obvious conclusion of anyone listening to the discussion at such a meeting would have been that the group felt itself inadequately informed. As a result, the writer, as chairman of the committee, undertook a study of the teaching of physics in the high schools of Cambria County, Pennsylvania. The hope was that this study, though of small scope but complete, would indicate directions of study and perhaps give some clues to basic weaknesses in high school teaching.

The questionary method was used (Table I). As usual, the response was only partial; however, with a small territory, a follow-up was possible until every high school was represented in the study. In a few cases the information is doubtful because of a certain reticence on the part of the individual interviewed. But these cases will, in general, be noted. There are in Cambria County 26 high schools. All but one have enrolments ranging between 100 and 700. This one high school has 1800 juniors and seniors enrolled; the lower years are given in three junior high schools.

Among the 26 high schools represented in this survey, only one, the largest, offers vocational work, in the sense of shop work. Commercial courses are usually available. However, the typical course is a college preparatory or liberal arts course, the latter term being used for courses not vocational or having the stated objective of college preparation. In this course the science requirement usually is a year of biology followed by a choice of chemistry or physics for the second year. These two courses are often alternated. The text is usually Millikan and Gale, Fuller-Brownlee, or Black and Davis.

The courses are often given without separate laboratory periods. If there is a noticeable tendency, it would seem to be toward a course given one hour daily with no individual laboratory work. Some schools have one double laboratory period and some have two; but they are few and perhaps decreasing in number. The larger schools especially seem to be dropping laboratory work. The typical teacher is overloaded and inadequately supplied with new equipment.

These general statements are only tentative since they are based on one group of high schools in a partly farming, partly industrial-mining community. Perhaps further study in other parts of Pennsylvania will alter the picture for the state.

The information concerning teacher training was perhaps the main purpose of the study, al-

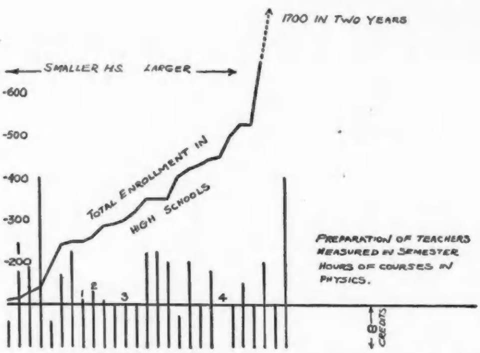

FIG. 1. Each vertical line represents the number of college semester hours credit in physics courses earned by the physics teacher in the corresponding high school. The arrangement is from left to right in order of total enrolment in the high school in which the teacher works.

though it may turn out to be a less determining factor for good teaching than is supposed. Fig. 1 represents the results. It is by no means true that the larger schools have the better prepared teachers. In fact, there seems no obvious relation between enrolment and teacher preparation. Even the teachers having little or no preparation are not confined to the small schools.

The numbered individuals in Fig. 1 need special mention. No. 1 did not answer definitely but perhaps had had some physics, possibly 8 hours, since he had studied some chemical engineering. No. 2 hesitantly claimed 10 hours but "felt the need of a review course in physics." No. 3 "does not remember whether he had 9 hours of chemistry and 6 hours of physics, or vice versa." A compromise 8 hours is plotted. No. 4 admits that he has never had a physics course.

The number of courses in physics on the transcripts of the teachers is no absolute index of the value of the teacher. In fact, one or two of the teachers, here shown with 8 credits, apparently did a fair job, while two with better preparation, who were busy with some administrative work, seemed to be doing very little effective teaching. On the whole, however, the preparation seems sadly inadequate.

Figure 2 is a summary of further pertinent information. The lower curve is repeated for reference. The middle curve shows a rising tendency, as one might expect. The excessive peaks probably represent faulty estimation. The better prepared teachers were the more conservative in estimation. Incidentally, there seemed to be not so much a lack of funds for apparatus as space for storage, the schools in general being very crowded.

The upper curve shows a slight downward trend in the larger schools, which may be authentic. In the smaller schools, the lack of vocational and industrial arts courses throws the whole student body into the liberal arts program. The organization of the curriculum tends, therefore, to demand either physics or chemistry in the last two years, and in some schools both may be elected. The enrolment in physics is very likely to be higher in a school with a limited offering of courses.

TABLE I. *Questionary to be filled out by teacher.*

NOTE: This information will be used purely for statistical purposes.

Name.................................... Address....................................
High School........................... Address....................................
Courses taught and number of sections of each in all subjects taught:
....................( ), ...( ), ...( ),
....................( ), ...( ), ...( ).
Text used in physics class..
Text used in physics lab...
Hours devoted to physics class per week..................................
Hours devoted to physics lab. per week....................................
Number enroled in high school..
Number enroled in physics..
Is physics elective or required?..
Is there any mathematics prerequisite?.......................................
If so, what?...
Is physics open to all or to a selected group?.............................
Is physics elected by a particular group mostly?.........................
If so, would you consider the group better than average students?...
Is physics enrolment in your school increasing or decreasing?..
Approximately what is the value of the equipment available in your school? $..
What is the approximate average annual appropriation for physics equipment? $...
How many semester hours in physics courses in college have you?...
What were these courses?........................, ...
........................, ..., ..., ...
....
When taken?.......................... Where?..........................
In your school what is the relative emphasis placed on physics as compared to biology and chemistry?
What attempts are made to meet needs of vocational students?
....
....

In general, it would seem that the position of physics in present-day high schools is not at all satisfactory. It is probably on the way out. Although it is undoubtedly dangerous to generalize from so little information, it does seem that certain factors in the situation stand out as conducive to such a trend.

The first factor is that physics is probably poorly taught, but not only because the teacher is poorly prepared, but because teaching loads are heavy. A load of six or seven hours a day, plus paper grading, extra curricular work, and the statutory requirement of further courses to improve preparation, does not leave adequate time for setting up and taking down apparatus. Physics without adequate laboratory and demonstration is necessarily weak. The course, therefore, is likely to seem unintelligible to those without adequate experience with apparatus, and to those who visualize poorly, it tends to become hopelessly confusing.

The second factor is perhaps the weakened emphasis on mathematics in some cases. Although mathematics is still taught, the course is becoming apologetic. Eventually, it may be on the way out too. Some of the teachers, especially in the school where vocational courses are given, have found it necessary to step down their physics courses to accommodate students who lack mathematics.

Another factor is the unpopularity of the course with administrators. Physics is expensive, requires disproportionate time and, if extra laboratory periods are given, complicates scheduling of classes. All of these things irritate administrators and tend to prejudice them against the subject. Since the tendency seems to be to place administration in the public schools in the hands of men who have had little or no training in humanities, science or letters, but almost solely in professional education, one perhaps should expect to see physics thought of in terms of its effect on the smoothness of the administrative machinery. Physics is being rated on the basis of its nuisance value.

It is entirely possible that the compelling factor which is driving physics out, if such appears to be in fact the tendency, is an underlying philosophy of education. Perhaps physicists as an organized group should interest themselves in

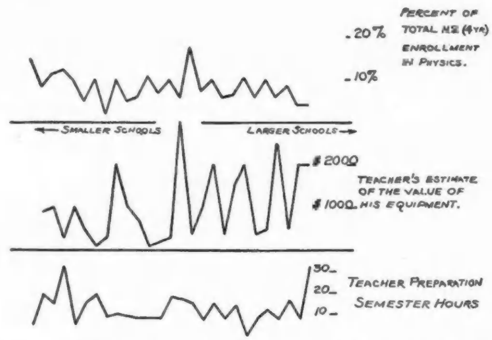

FIG. 2. Relationships between size of high school and physics enrolment and equipment.

the general problem of education, especially in the high school. Quite obviously, physics can defend itself in any fair field; but the momentum of a slow movement may crowd it out of the picture with no opportunity to defend its case. The present tendency to replace it by some pale, apologetic, Alice-in-Wonderland type of social science course or a general science "survey" course offering material chosen for its ability to capture a supposedly vagrant interest, is hardly likely to impress the physics teacher as a step forward.

If movements are on foot that will result in the disappearance of science and mathematics from the curriculum, then the physicists should be aware of them. What to do may not be at all clear, but, in any case, the first step is to collect adequate information. At present, this study is being followed up in other counties of Pennsylvania by the other members of this committee, Professor J. A. Swindler, Westminster College, Professor Chas. O. Williamson, Carnegie Institute of Technology and Professor W. N. St. Peter, University of Pittsburgh. Similar studies are to be made in West Virginia by another committee.

It would be advisable to have similar studies made in several other states. A random scattering of completely covered counties should yield much information of a suggestive nature. This committee would welcome any information of this type. It should be emphasized that if anyone is interested in making a study, he should make it quite complete for a given geographic unit. Obviously, the most significant information is the most inaccessible.

The Diffraction of Light by Supersonic Waves in Liquids; Apparatus for Demonstration and for an Intermediate Laboratory Experiment

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IN 1922 Brillouin[1] predicted that if a liquid were penetrated by progressive waves of compression of short wave-length and at the same time were irradiated by light, there would be diffraction of the light by the regular pattern of density-variations in the liquid. The phenomenon was demonstrated during 1932 by Debye and Sears[2] in the United States and by Lucas and Biquard[3] in France. Both pairs of investigators passed light through a linear slit and through a trough of liquid containing supersonic waves generated by a quartz crystal. The diffraction patterns were recorded on a photographic plate. A study of the diffraction patterns, together with a few other easily made measurements, permits determination of the speed of the supersonic waves and of the adiabatic compressibility of the liquid. The present paper describes a simplified apparatus which is suitable for lecture demonstration experiments, and by which reasonably accurate laboratory measurements can be made.

The essential elements are shown in Fig. 1. A table spectrometer of the kind used for optical experiments in the first-year physics laboratory provided the collimator, telescope and scales for measuring angles. A sodium vapor lamp served as a source of monochromatic light. The glass trough rested on a brass plate which was bolted to the prism table. The trough was $3\frac{1}{2}$ in. long and 2×2 in. in cross section. It was made of good quality window glass cemented with a liquid porcelain sold under the trade name "Insalute." This cement is impervious to acids and to the organic liquids on which measurements were made by the authors.

When electrically conducting liquids were studied, an inner cell containing xylol or toluene was placed inside the glass trough. With this arrangement the crystal can vibrate in a non-conducting liquid and the supersonic compressional waves pass through a thin mica window into the outer liquid. The inner cell was made of copper and measured $2\frac{1}{2}\times1\frac{7}{8}\times\frac{7}{8}$ in., open at the top, and with a 1×1-in. mica window. The sealing agent was Duco cement, and the cell was painted with Eastman Kodacoat acid-proof paint.

The X cut quartz crystals, which had fundamental frequencies near 7 megacycles/sec, were silvered by a deposition method. Electrical contact was made to the crystal through the jaws of a clamp mounting, one jaw being a piece of stiff brass, the other a safety razor blade. The tension was considered to be optimum when it was barely enough to hold the crystal and to make good electrical contact. The mounting brackets for the inner cell and for the crystal clamp were made so that adjustments were possible in three dimensions.

The radiofrequency generator was a 210 triode in a self-excited tuned-plate tuned-grid oscillator circuit. Ample output was secured with 20-w

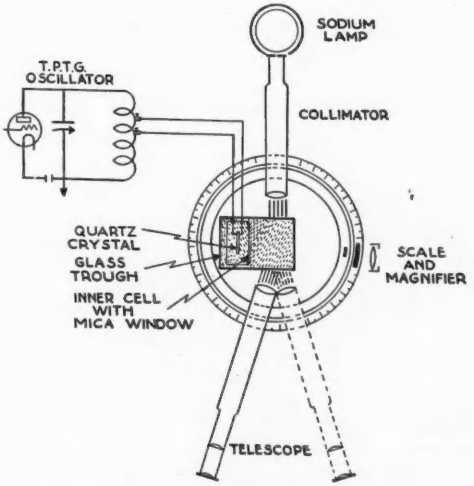

FIG. 1. Apparatus for study of the diffraction of light by supersonic waves in liquids.

[1] L. Brillouin, Ann. de physique **17**, 103 (1922).
[2] P. Debye and F. Sears, Proc. Nat. Acad. Sci. **18**, 409 (1932).
[3] R. Lucas and P. Biquard, Comptes rendus (Paris) **194**, 2132 (1932).

input power. The plate coil had 10 turns, $2\frac{1}{2}$ in. in diameter and 7 in. in length. Normally, the lead wires to the crystal were clipped across three of the ten turns of the coil.

For demonstration purposes, a low-power lens can be substituted for the telescope and the diffraction pattern focused on a screen for observation by an audience. It is possible to secure a focus without the low-power lens by adjusting the collimator to produce a converging beam of light. Vivid effects can be secured by using white light or the light from a mercury source.

TABLE I. *Supersonic speeds and adiabatic compressibilities in several liquids by the piezometer (P) and light diffraction (L) methods.*

Temp. (°C)	Speed (m sec^{-1})	Adiabatic Compressibility ($\Delta V/V$ atmos^{-1})	Method	Observer
		Benzene		
25.0		69.1	*P*	Tyrer
25.0	1299	69.1	*L*	Authors
		Carbon Tetrachloride		
25.0		75.3	*P*	Tyrer
25.0	923	74.7	*L*	Authors
		Toluene		
25.0		69.9	*P*	Tyrer
25.0	1290	70.9	*L*	Authors
		Xylol		
25.0		68.0	*P*	Tyrer
25.0	1317	68.2	*L*	Authors
		Water		
29.2		45.9×10^{-6}	*P*	Rama Rao
25.0	1499	45.3	*L*	Authors

MEASUREMENTS

The following relations are used:

$$n\lambda=\lambda'\sin\theta, \tag{1}$$

$$v=f\lambda', \tag{2}$$

$$B_a=1.013\times10^6/\rho v^2, \tag{3}$$

where λ is the wave-length of the light, θ is the scattering angle for any order, n has integral values, λ', f, and v are, respectively, the wave-length, frequency, and speed of the supersonic waves in the liquid under investigation, and ρ and B_a are the density and adiabatic compressibility of the liquid; B_a is given as contraction in unit volume per atmosphere. The usual procedure is to determine θ for any observed order n, make measurements of ρ and f, the latter with a frequency meter or wave meter, and then calculate the speed of the supersonic waves and the adiabatic compressibility of the liquid.

TABLE II. *Typical measurement—benzene.*

Temp. (°C)	Order No.	Angle of Diffraction (min)	Supersonic Wave-Length (cm $\times10^2$)	Supersonic Speed (m sec^{-1})
25.5	2	23.0	1.760	1297
25.9	2	23.0	1.760	1297
26.0	3	34.4	1.768	1300
26.1	2	23.1	1.750	1289
26.0	3	34.8	1.747	1287
27.3	3	34.9	1.740	1281

Wave-length of light, 5893×10^{-8} cm;
Supersonic frequency, 7.36 megacycle sec^{-1};
Density at 25°C, 0.870 gm ml^{-1};
Adiabatic compressibility 69.1×10^{-6} atmos^{-1}.

The length of path traversed by the supersonic waves was varied by sliding the glass trough on the brass platform until the maximum number of orders in the diffraction spectrum could be observed. The number of observable orders may be increased by applying higher power to the crystal, but caution is necessary to avoid instability or fracture of the crystal. Measurements were made in several organic liquids and in aqueous solutions of acetic acid. In measurements on xylene and toluene nine orders were usable, but only three could be used in the case of carbon tetrachloride and benzene. For the determinations with aqueous solutions of acetic acid, where the inner cell was inserted in the glass tank, the second order was the highest usable for the denser solutions and the third order for solutions of lower concentration. For two orders the probable error in measuring the angle of diffraction was 2.2 percent, but for nine orders this probable error amounted to only 0.2 percent. Temperature was measured by means of a thermometer placed near the light beam in the liquid. The values of supersonic speed in a liquid were plotted as a function of temperature, and a straight line was drawn through the plotted points. The speed at 25°C was read from the curve.

Table I contains the results of measurements of a few liquids, together with values published by other observers. A sample measurement is given in Table II. The authors' results for aqueous solutions of acetic acid were reported elsewhere.[4]

[4] J. Chem. Phys. 7, 162 (1939).

The accuracy of results obtained with the apparatus described is, of course, limited by the error introduced in reading the diffraction angles directly from the spectrometer scale, and in the determinations of frequency, density and liquid temperature. Also, the ideal conditions for which Eqs. (1), (2) and (3) apply do not exist in practice. However, there is excellent agreement between observations made by the means described here and the measurements of other workers, and the accuracy is sufficiently good for the system to have merit as a teaching aid.

Unique Oscillographic Demonstrations

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AND

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THE ever increasing popularity of the oscillograph as a medium for classroom demonstration emphasizes the desirability of extending the range of this instrument to include extremely slow and nonrecurrent phenomena. The limitations of the cathode-ray oscillograph as regards linearity of time axis for sweep velocities of 5 cm/sec or less, and the complexity of circuit design for base-line shifting are too well known to require discussion here.

THE OSCILLOGRAPH

These limitations may be overcome, as was recently demonstrated,[1] by the use of a moving phosphorescent screen oscillograph having a mechanically driven celluloid belt coated with long persistence phosphorescent zinc sulfide. A luminous tracing is produced on this screen by a small, sharply defined, intense beam of light reflected from the mirror of a heavily damped, short period galvanometer. The tracing has approximately the same brilliance and definition as that of the cathode-ray tube, but differs in that it remains visible in a darkened room for nearly a minute, the persistence period being dependent upon the choice of luminescent coating, as will be explained below. An earlier form of this instrument has been described,[2] but subsequent improvements and simplifications deserve brief mention here.

The present form of the instrument is rectangular and, in the interests of portability, has been built into a carrying case measuring $5\frac{1}{2}\times18\times20\frac{1}{2}$ in. over-all, as shown in Fig. 1. For convenience in making adjustments the parts embodied in the instrument are attached to a removable base. The moving screen passes over four hollow metal drums which form the corners of a rectangle, as shown in Fig. 2. The left-hand rear drum drives the belt in a clockwise direction; it is in turn driven through bevel gears by an electric phonograph motor. The right-hand rear drum is mounted on a movable arm to which a spring is attached and serves to maintain tension on the belt. The two remaining drums are idlers and simply guide the screen past the viewing aperture. All drums must be painted with clear lacquer to prevent blackening of the belt by metal rubbed from the drums.

All other essential parts are located within the enclosure formed by the phosphorescent belt. The galvanometer[3] is mounted in the right-hand

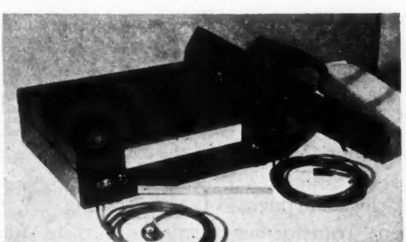


FIG. 1. Demonstration form of moving phosphorescent screen oscillograph in carrying case.

[1] Hoecker, Am. J. Phys. (Am. Phys. Teacher) 7, 261 (1939).
[2] Hoecker and Asher, Rev. Sci. Inst. 9, 148–150 (1938).
[3] Built by The Sanborn Co., 39 Osborn St., Cambridge, Mass.

rear corner (Fig. 2) with the coil suspension horizontal; it is of the taut-suspension, D'Arsonval type with a period of approximately 1/50 sec. In terms of deflection at the moving screen, the galvanometer has a sensitivity of 20 millivolt/mm; it has a current sensitivity of 15 μamp/mm and a coil resistance of about 1300 ohms. This galvanometer differs from the one described in the earlier article[2] in that it has a larger mirror and a flat, rectangular form which simplifies mounting. The face-aluminized concave mirror, focal length 8 cm, diameter 0.5 cm, forms an image of the horizontal, spiral filament of the tungsten exciter lamp on the cylindrical lens located at its focal distance from the screen just to the right of the viewing aperture. The 7.5-v, 10-amp exciter lamp is completely enclosed in a metal housing except for a small aperture through which the light emerges to the galvanometer mirror. The filament of this lamp is a single compact spiral, forming a rectangular source approximately 0.1×0.6 cm. It is important to note that this arrangement produces an energizing spot elongated in the direction of the galvanometer deflection and hence does not

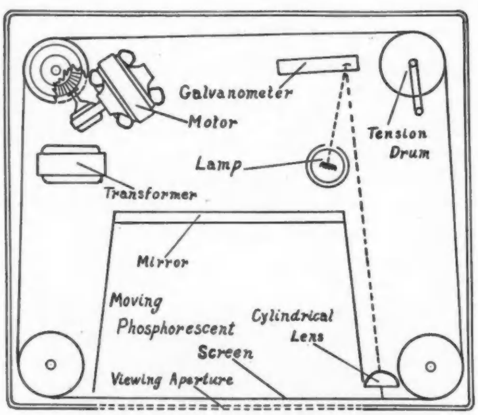


FIG. 2. Diagram of the oscillograph.

spoil the sharpness of the tracing. A small filament transformer furnishes current for the exciter lamp. A control which rotates the galvanometer suspension permits adjustment of the base line to any vertical position on the screen.

The moving phosphorescent screen is made of a single strip of clear celluloid with ends

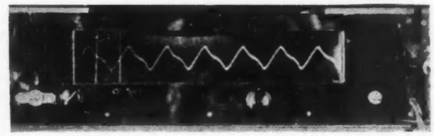


FIG. 3. Distorted wave form produced by the pole of a magnet oscillating through a coil.

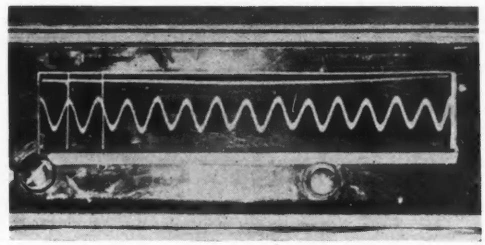


FIG. 4. Sinusoidal wave form produced by the center of a magnet oscillating through a coil.

cemented to form an endless belt 7 cm wide and 150 cm long. Before the ends of the celluloid strip are cemented, the active material, phosphorescent zinc sulfide, is applied in the form of a suspension in clear lacquer by means of a soft brush. Proper choice of the phosphorescent material with regard to (*a*) intensity of luminescent emission,[4] (*b*) periods of excitation and decay,[5] and (*c*) color sensitivity[6] of the eye is of extreme importance. Green phosphorescent zinc sulfide was used in earlier experiments in conformity with (*c*) above. However, experience has shown that the effects of (*a*) and, more particularly, (*b*) are of greater importance in achieving maximum brilliance of the tracing and high contrast against the background. Best results are obtained for all purposes by using orange or red phosphorescent zinc sulfide.[7]

When used for class demonstrations the screen is viewed, as shown in Figs. 3 to 7, in a darkened room. If desired, a single observer may use the instrument in a well-lighted room by lifting the lid slightly and viewing the back of the screen

[4] Lewschin, Acta Physica Polonica **5**, 301–317 (1936).
[5] Byler, J. Am. Chem. Soc. **60**, 632–639 (1938).
[6] Houstoun, *A treatise on light* (Longmans, Green, 1928), p. 350.
[7] Pfaltz and Bauer, Empire State Bldg., New York, N. Y., or Hammer Laboratories, Denver, Colo.

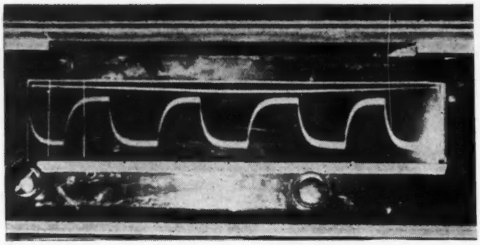
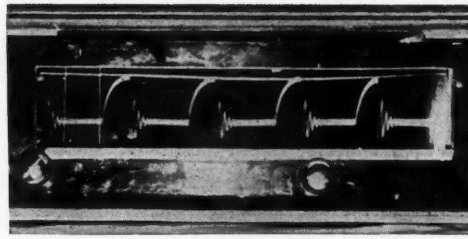


FIG. 5. Curves of growth and decay of current in an inductive circuit.



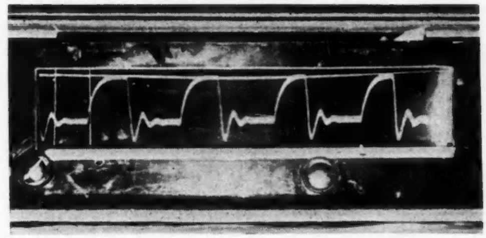


FIG. 6 AND 7. Curves showing charge and oscillatory discharge of a condenser in an inductive circuit.

in the mirror shown in Fig. 2. When used in this manner the front viewing aperture must be covered.

Physics Classroom Uses

The scope of an experiment in electromagnetic induction—for example, the coil and magnet[8]—is greatly extended, if, instead of connecting the coil to a demonstration galvanometer, it is connected to the instrument described. The demonstration galvanometer does, of course, show an impulse when the magnet is thrust into the coil, and indicates the direction of the current, but gives no idea of the true relation between the generated emf and relative position of the magnet in the coil. When the coil is connected to the galvanometer of the moving screen oscillograph, a curve is plotted on the screen showing very clearly the relation between induced emf and time as the magnet is inserted into or withdrawn from the coil. Of course, it is essential that the plotted curve embody no distortion due to inertial or damping characteristics of the oscillograph galvanometer. The galvanometer described here has been specially designed to register electrocardiac impulses without distortion. On the basis of the manufacturer's statement regarding tests on wave forms of known shape, it is believed no distortion is present below frequencies of 50 cycles/sec.

For use in connection with this new type of oscillograph the classical coil and magnet experiment has been somewhat extended by arranging the magnet to oscillate within the coil. A small coil of several hundred turns of fine wire wound on a thin fiber spool is mounted movably on the vertical rod of a tripod. A round bar magnet is suspended vertically between coil springs suitably chosen with regard to the mass of the magnet and the desired period of oscillation. By vertical adjustment of the coil any desired section of the magnet may be caused to oscillate through it.

When the end of the magnet oscillates with respect to the center of the coil a distorted wave form is plotted on the screen, as shown in Fig. 3. This is, of course, a simple harmonic repetition of the classical experiment. When the center of the magnet oscillates with respect to the coil, a very close approximation to a sinusoidal waveform is plotted on the screen (Fig. 4). Analysis of the manner in which the lines of force are cut by the coil in these two cases discloses that the small "hump" on the right-hand slope of the distorted wave develops into a full-fledged crest of the sinusoidal curve, bringing about a frequency doubling effect, as is shown by comparison of Figs. 3 and 4. Obviously the period of the magnet and hence that of the wave form may be adjusted to any desired value by the addition of weights to the magnet. Clearly the student may observe that the magnet is oscillating with simple periodic motion. Conclusions

[8] Sutton, *Demonstration experiments in physics* (McGraw-Hill, 1938), E-216, p. 339.

to be drawn from these illustrations with regard to uniformity of field distribution about a bar magnet are a matter of personal choice. Applications to the study of alternating currents are obvious, and the authors know of no simpler means of correlating visually simple periodic motion with the sinusoidal wave form.

As further illustration of the usefulness of the moving screen oscillograph, Fig. 5 demonstrates the growth and decay of current in an inductive circuit. Here again the advantages of this type of demonstration lie in lengthening the growth and decay periods by adjustment of the time constant of the circuit. The student actually sees the curve plotted as the switch in the test circuit is opened and closed.

Figures 6 and 7 show the interesting case of the charge and damped oscillatory discharge of a condenser in an inductive circuit. The period of the oscillations was varied by adjusting the capacitance of the condenser. The further interesting case, not shown here, is that in which the capacitance is made sufficiently large so that the charge leaks off the condenser very slowly; that is, the exponential terms become real. A leakage period of nearly 2 sec has been demonstrated in this manner. When these demonstrations are performed in a darkened room, the switch in the test circuit should be illuminated so that it is visible to the group. This may easily be accomplished by using a small flashlight.

Biophysical Demonstrations

This oscillograph also conveniently demonstrates physiological potential differences, either recurrent or nonrecurrent. Most electrical impulses of physiological origin are extremely feeble; those due to the heart muscle usually do not exceed 1 millivolt and, hence, must be amplified before they can be demonstrated. Any amplifier with a gain of at least 250 and with good low frequency response will serve. The circuit diagram,[9] Fig. 8, of a single stage amplifier which has been used for this purpose is notable only because of circuit constants.

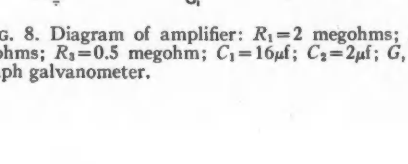


FIG. 8. Diagram of amplifier: $R_1=2$ megohms; $R_2=3$ megohms; $R_3=0.5$ megohm; $C_1=16\mu f$; $C_2=2\mu f$; G, oscillograph galvanometer.

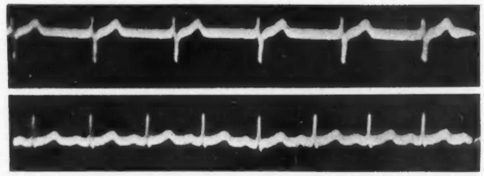


FIG. 9. Electrocardiac emf.

Figure 9 shows the type of tracing obtained in demonstrating the electrical behavior of the heart. Other currents of action, such as skeletal and alimentary muscle contractions, either voluntary or stimulated, may be extensively studied and demonstrated. Abnormal muscle tremors, such as those seen in the anxiety states, and nerve disorders, as well as the effects of exercise, fatigue and change of physiologic state, have been shown and studied as a continuous observation on the moving phosphorescent screen.

Electrical connection to the subject is effected by a small metallic plate strapped to each arm or other portions of the body. When prepared salt jelly is not available, good electrical contact is secured by placing a small pad of cloth saturated in ordinary salt solution between the metallic plate and skin.

All illustrations shown here are actual photographs of wave forms produced in the manner described, though we feel compelled to admit that they are double exposures. The two white lines at the extreme left of the viewing aperture represent approximately 0.5 sec on the time axis.

The authors wish to express their appreciation for suggestions made by Professor Paul Kirkpatrick of Stanford University regarding the preparation of the manuscript.

[9] Private communication from Dr. George Walker, University of Kansas Medical School.

NOTES AND DISCUSSION

Protective Device for the Synchronous Clock

SOME time ago the use of the self-starting synchronous clock as a convenient and economical laboratory substitute for the stopwatch was reported in this Journal.[1] If this clock is accidentally connected to a 110-v d.c. line, it will burn out. A simple way to protect it is to install in series with the clock a capacitor, or a capacitor and resistor in series. The capacitance and resistance must be so chosen as to allow the proper current in the clock. With our clocks[2] we use a 0.5-μf 600-v paper capacitor (electrolytic capacitors must not be used) and a 1250-ohm, 2-w resistor, the combined cost being about 50 cts. A 0.4-μf capacitor with no resistor would have served, but this size was not available in a single unit. The selection of suitable values for the capacitance and resistance is easily made if one has available a decade capacitor, a resistance box and an a.c. milliammeter. First the current for normal operation of the clock is measured and then capacitance, or resistance and capacitance, added in series until the current is again the normal value. If a suitable milliammeter is not available a rectifier type voltmeter may be connected across the clock instead. It is interesting to construct the vector diagram showing the relation of the reactances and total impedance. The characteristics of our clocks[2] were approximately as follows: $i=30$ ma, $Z=3670$ ohms, $\omega L=3220$ ohms, $R_{\text{a.c.}}=1760$ ohms, $R_{\text{d.c.}}=740$ ohms. The addition of the 1250-ohm resistor and 0.5-μf capacitor ($1/\omega c=5300$ ohms) gives a total impedance of 3670 ohms as before, but the current is "leading" rather than "lagging."

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CHAS. WILLIAMSON

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[1] Am. J. Phys. (Am. Phys. Teacher) 5, 41 (1937).
[2] Model 2FO1 Telechron.

Rule of Signs for Lens and Mirror Equation

THE need for a uniform rule of signs for the lens and mirror equation and the present state of confusion which exists in practically all of the physics textbooks has been described by R. B. Abbott.[1] The writers of physics textbooks have had to choose between one equation for both lenses and mirrors with different rules of signs and different equations for lenses and mirrors with a single rule of signs. Abbott recommended different equations with a single rule of signs based on a rectangular coordinate system. The A. A. P. T. Committee on the Teaching of Geometrical Optics recommended, after consideration of the *Report on the Teaching of Geometrical Optics*[2] published in 1934 by the Physical Society of London, that the rule of signs for lenses be as follows: Object or image distance is positive (or negative) if the object or image is real (or virtual).[3] The focal distance f for a convex lens is positive and for a concave lens is negative.

The first part of this rule would apply also to the mirror equation which would have the same form as the lens equation, but the second part would be changed to state that f for a concave mirror is positive and for a convex mirror is negative. This requires a knowledge of the meanings of *concave* and *convex*, and adds confusion because the signs are opposite for lenses and mirrors. From the points of view of teaching and general use it would be advantageous to have a single equation and a single rule of signs to apply to all cases for both lenses and mirrors. Such a single equation and single rule are:

$$\frac{1}{s_1}+\frac{1}{s_2}=\frac{1}{f},$$

where s_1 is the object distance, s_2 is the image distance and f is the focal length; and the signs are determined by the statement that *anything real has positive distance* and *anything virtual has negative distance*.

In the application of this rule it is seen that s_1 is + (or −) if the object is real (or virtual), s_2 is + (or −) if the image is real (or virtual), f is + (or −) if the principal focus is real (or virtual) and the rule applies equally well to lenses and to mirrors.

This rule of signs applies also to the lens equation,

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right),$$

where n is the index of refraction and r_1, r_2 are the radii of curvature of the surfaces. Applying the rule, f is taken as + (or −) if principal focus is real (or virtual); r for a surface is + (or −) if this surface opposite a plane surface produces a real (or virtual) principal focus.

In applying the rule to determine the sign of r, it must be recognized that the surface whose radius is r is determined not only by its radius but by whether or not it is concave or convex with respect to the air. It is easy to recognize whether or not any surface when opposite a plane surface will give a real or virtual principal focus.

The rule of signs recommended here may be briefly stated as: *if real* +; *if virtual* −.

This rule may have been proposed previously but it has not received the adoption to which it would seem to be entitled. Of ten popular physics textbooks examined by the writer, only one gave this rule as applying to the simple lens equation, and it gave a different rule for mirrors. The application of the rule requires simply that the student know how to distinguish real from virtual images, objects and principal foci. It has been found to simplify and aid the teaching and use of the lens and mirror equation.

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[1] Abbott, Am. J. Phys. (Am. Phys. Teacher) 4, 23 (1936).
[2] Review of report, Am. J. Phys. (Am. Phys. Teacher) 3, 140 (1935).
[3] A.A.P.T. Committee, Am. J. Phys. (Am. Phys. Teacher) 6, 78 (1938).

The Nature of Sliding Friction

IN some recent experiments on sliding friction, J. J. Bikerman and E. K. Rideal[1] find that the frictional force is proportional to the normal force and returns reversibly to its initial value after the addition and subsequent removal of another normal load. They interpret this result as proving that "cohesion, welding and plastic flow are eliminated as factors in the friction," and go back to Coulomb's old explanation that surface irregularity is the sole cause of sliding friction. It seems clear that a different interpretation of these experiments is possible—one which makes them more in accord with the work of other investigators.

For intermediate loads between surfaces in air, the presence of adsorbed oxygen and oxide films prevents the proper flux conditions for welding between the surface contacts and probably neutralizes the surface fields sufficiently to make adhesion negligible. Under these conditions the energy used to overcome friction is expended in local melting and plastic flow of *each surface separately* (or perhaps of only one surface). Thus there is no welding or adhesion between the surfaces, and the observations of Bikerman and Rideal are at once explained. That local melting and plastic flow play a predominant part in frictional forces is amply proved by the brilliant measurements which have been made of contact temperatures during sliding[2] and the change of electrical spreading resistance with normal load.[3]

As the normal force increases, a condition is finally realized where the adsorbed films of "welding inhibitor" are less effective and bridges[4] are formed between the surfaces, the breaking of which in shear is an important means of energy dissipation. Finally, seizing of the surfaces may occur. In no case for surfaces actually used in machinery can the interlocking of surface irregularities be more than a minor factor in the total frictional force. Very strong evidence of this is given by the recent work of F. P. Bowden and T. P. Hughes[5] showing that friction is greatly increased between completely outgassed surfaces under vacuum conditions, as compared with the same surfaces in air. Such treatment cannot produce any important change in surface irregularities. Another point which supports the local melting process and not the surface interlocking hypothesis is the great influence of thermal conductivity in the performance of certain industrial bearings.[6] A thin bearing surface supported by a good conductor, such as copper, and another supported by a poorer conductor, such as steel, differ markedly in actual performance. Thermal conductivity could hardly be of importance if the mechanism proposed in reference 1 were the principal one operative.

All the experiments support the following picture for the sliding friction between resonably smooth surfaces in air. At low loads, local melting, polishing and plastic flow occur on each surface at the points of greatest pressure (or on only one surface if the melting points differ greatly). Practically no "bridges" are formed by welding between surfaces because of the presence of adsorbed oxygen or oxide layers which neutralize the surface fields of the metals.

With large loads, welding occurs between the surfaces at the points of highest pressure, and one or both surfaces begin to scar owing to the tearing out of metal. For the welding process to begin, the pressure must be high enough to destroy the protecting films of oxide, adsorbed oxygen or boundary lubricant. That welding is more important than surface adhesion may be inferred from some recent observations of Bowden and Leben.[2] They find that the kinetic friction is greatly lowered by the presence of adsorbed oxygen, while adsorbed nitrogen or hydrogen produces no change. All three gases should be equally effective in lowering the adhesive forces of the atoms exposed at the surfaces. It appears, therefore, that the most important property of a boundary lubricant is not film strength but its chemical effectiveness as a welding inhibitor.

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1 Bikerman and Rideal, Phil. Mag. **27**, 687 (1939).
2 Bowden and Leben, Proc. Roy. Soc. **169**, 371 (1939).
3 Bowden and Tabor, Proc. Roy. Soc. **169**, 391 (1939).
4 Adam, *The physics and chemistry of surfaces* (Oxford, 1938), p. 220.
5 Bowden and Hughes, Proc. Roy. Soc. **172**, 263 (1939).
6 R. S. Shankland, unpublished studies of bearing surfaces.

The Bernoulli Theorem

IN an article on the Bernoulli theorem, G. A. Van Lear[1] forcefully criticizes the misconception of the so-called "pressure energy per unit volume" of an incompressible fluid in streamline flow and gives an interesting and perfectly sound elementary derivation of the Bernoulli theorem. Near the end of the article he rewrites the theorem in the usual form,

$$P+\rho gh+\tfrac{1}{2}\rho v^2=\text{const},\qquad (1)$$

and points out that the equation holds for all points of a single flow tube. In the next paragraph, he says: "Some insight into the reason for the persistence—and usefulness—of the 'pressure energy' concept may be gained by reflecting that, when such a term is added to the kinetic and gravitational energy terms, the result gives the total work which will be available per unit volume of liquid *when it emerges* from the section of flow-tube. The fallacy lies in crediting each particular unit volume with that much energy before emergence, for the term in question represents work which will be done on it by the liquid behind it *as it emerges*."

It should be observed that the difference between the energy per unit volume of fluid just before, during and just after emergence from the section of flow-tube is strictly fictitious; that is, the difference approaches zero as ΔV approaches zero. Thus, although it is easy to misinterpret the elementary derivation and conclude that the limit (as $\Delta V\rightarrow 0$) of the work per unit volume done on the volume element ΔV *as it emerges* from the section of flow-tube is finite and equal to the pressure at the place of emergence, the work actually done on it per unit volume is zero in the limit ($\Delta V\rightarrow 0$); for, in addition to the pressure of the liquid behind it as it emerges, there is also the pressure of the liquid before it pushing in a direction opposite to the direction of its motion—and this latter pressure differs from the former by an amount which approaches zero as ΔV ap-

proaches zero, thus making the net forward force infinitesimal. It appears to the writer that this confusion results from the attempt to obtain detailed information from an elementary derivation which is incapable of giving it, even though the derivation is perfectly sound in principle and final result. The details of the process can be understood only by application, either qualitative or quantitative, of the calculus. Since the general derivation of the Bernoulli theorem in advanced texts and treatises requires a knowledge of vector analysis, the following derivation, which presupposes only a knowledge of elementary calculus and the Taylor expansion, may be of value to intermediate students and even to sophomore students if enough explanation accompanies it.

Let s denote the distance measured along any particular line of flow of an incompressible fluid in ideal streamline flow. The origin will be arbitrary. Let h denote the height of any point on the streamline above an arbitrary zero level. Consider a small volume of fluid, $\Delta V=\Delta A\Delta s$, where ΔA is the cross-sectional area at point s of a small tube of flow which surrounds our chosen line of flow. Since both ΔA and Δs are functions of s if we fix our attention on a *particular* small volume of fluid, let us remind ourselves of this by writing

$$\Delta V=\Delta A(s)\Delta s(s).$$

By expanding the pressure $P(s)$ about the point s in a Taylor series and neglecting terms beyond the second in the series, we find the work done by the fluid surrounding our particular volume ΔV as it is moved from s_0 to some other point s_1 to be approximately

$$\Delta W=-\int_{s_0}^{s_1}(dP/ds)\Delta s(s)\Delta A(s)ds.$$

The work per unit volume, dW/dV, is however, exactly

$$\lim_{\Delta V\to 0}\frac{\Delta W}{\Delta V}=\lim_{\Delta V\to 0}-\left(\frac{1}{\Delta V}\right)\int_{s_0}^{s_1}\left(\frac{dP}{ds}\right)\Delta s(s)\Delta A(s)ds$$
$$=-\int_{s_0}^{s_1}dP=P(s_0)-P(s_1).$$

By conservation of energy, dW/dV must equal the sum of the changes of gravitational and kinetic energy per unit volume between s_0 and s_1; therefore,

$$P(s_0)-P(s_1)=\rho g(h_1-h_0)+\tfrac{1}{2}\rho(v_1^2-v_0^2);\qquad(2)$$

and this is the Bernoulli theorem. It seems to the writer that when the theorem is derived in the foregoing manner, any concept of "pressure energy" is avoided and the physical significance of the theorem is sharply emphasized. By rearranging terms in Eq. (2) and dropping the subscript "1," the Bernoulli theorem can be written,

$$P(s)+\rho gh+\tfrac{1}{2}\rho v^2=P(s_0)+\rho gh_0+\tfrac{1}{2}\rho v_0^2.$$

With the theorem in this form it is at once clear that, if we choose point s_0 at a place where the pressure is zero, the sum of the three terms on the left, each evaluated at the point s, is equal to the sum of the gravitational and kinetic energy densities at the point s_0. This means that $P(s)$ is equal to the work which will be done on unit volume of fluid by the surrounding liquid as the liquid moves from any point s to a point of zero pressure. Van Lear, in a more recent article on "pressure energy,"[2] states this same result with further precaution by emphasizing the fact that the Bernoulli theorem applies only to the case of *steady-state* streamline flow, that is, streamline flow in which P is a function of s alone and not of s and the time. He further points out that, in strict mathematical analogy with our concept of gravitational potential energy per unit volume, one may correctly regard $P(s)$ as the pressure energy per unit volume in the case of *steady-state* streamline flow. This fact, as he stresses, by no means exonerates those who for years have spoken of "pressure energy" either in the naive and incorrect sense that attributes elastic energy to an incompressible fluid[3] under pressure or without making clear the physical relations between change of pressure energy per unit volume and the work done on unit volume by the surrounding fluid as the liquid moves from one place to another. The writer is in complete agreement with Van Lear when he states in his 1938 article: "It is hoped that future textbooks will not present the 'pressure energy' idea in its usual misleading form. Since the idea has for its chief virtue a slight abbreviation of treatment—an abbreviation which would be more than offset by the explanation necessary to a clear understanding of the matter—this would perhaps mean its complete disappearance." Furthermore, as Van Lear has emphasized, from a theoretical point of view, this concept of pressure energy is quite meaningless and useless when we turn our attention to problems of nonsteady-state flow; whereas, if we forget about pressure energy entirely and think in terms of the work done on unit volume by the surrounding fluid, we can satisfactorily solve problems in steady-state or nonsteady-state flow with the same physical approach. This can be illustrated by the following simple example of nonsteady-state streamline flow in which one is led to an incorrect result if an attempt is made to use the concept of pressure energy. Imagine a tall, vertical, stationary cylindrical tube containing a column of water of convenient height h resting on a piston which is moving downward with constant speed v; under these conditions the pressure distribution in the liquid is the same as if the piston were stationary. The water is doing work on the piston, at the rate Mgv where M is the mass of liquid and g, the acceleration due to gravity. At an arbitrary zero of time let a downward acceleration equal to Kt be imparted to the piston. This functional form for the acceleration has been selected merely for the sake of concreteness and does not limit the generality of the argument. When enough time has elapsed so that $Kt=g$, the liquid will be in free fall and the hydrostatic pressure throughout the liquid will be zero. Since at, $t=0$, the pressure was higher near the bottom of the liquid than at the top, anyone who unwisely applies the pressure energy concept will conclude that more work has been done by the surrounding fluid on unit volume of fluid near the bottom of the moving column of liquid between $t=0$ and $t=g/K$ than has been done during the same interval on unit volume near the top. That this conclusion is false can be seen directly by considering the work done on unit volume by the surrounding liquid. Since the column of water moves as a single body without change of shape during its descent, every volume element of the liquid undergoes, at each instant of time, the same acceleration as every other volume element. And

if we imagine all the volume elements to be disk-like and of the same mass, it is clear that each of these volume elements—moving under the influence of equal gravitational forces—must also be moving at each instant of time under equal net forces exerted by all the surrounding fluid. Also, of course, all volume elements move exactly the same distance between $t=0$ and $t=g/K$. Therefore, it follows that the work done on unit volume by the surrounding liquid between $t=0$ and $t=g/K$ is independent of the location of the volume element with respect to the bottom and top of the moving column of water. In the example considered the work done by the surrounding fluid is negative in the sense that the surrounding fluid prevents every volume element from achieving the free fall acceleration during the period from $t=0$ to $t=g/K$. The sum (integral) of all these negative amounts of work done on all the volume elements by the surrounding fluid between $t=0$ and $t=g/K$ must represent the total work done by all of the liquid on the piston during this period. That this is so can be proved by an application of the calculus.

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[1] Am. J. Phys. (Am. Phys. Teacher) **2**, 99 (1934).
[2] Am. J. Phys. (Am. Phys. Teacher) **6**, 336 (1938).
[3] As an additional blow against this completely incorrect way of conceiving of pressure energy, it is perhaps worth noting that the negligible amount of elastic energy per unit volume possessed at ordinary pressures by an almost incompressible liquid, such as water, will not be proportional to the pressure but to the square of the pressure, assuming that Hooke's law holds.

A Switch for Stopclocks

Following the suggestion of W. H. Michener,[1] we replaced stopwatches in our general physics laboratory with self-starting synchronous clocks. However, we had some difficulty installing a conveniently located switch that would be positive and quick-acting in starting and stopping the clocks. Professors Michener and Williamson of the Carnegie Institute of Technology inform me that with their clocks they use a separate 3-ft length of cord with a pear-shaped switch at the end. This switch is a tumbler type, H 271 Hemco cord switch, 6A 125V, The Bryant Electric Company, Bridgeport, Connecticut.

Another solution of the switch problem is shown on the clock in Fig. 1. It consists of an ordinary small toggle switch, the knob of which is turned down in a lathe to the same size as the shaft and then threaded. The little bracket on the lever arm is drilled and tapped, screwed onto the switch shaft, and held in place by a lock nut. The switch is installed in the clock by drilling a hole in the top of the wooden case. Two bumpers from the "dime" store complete the job.

The lever arm which throws the switch is large and easily located so that the student's attention need not be drawn from the apparatus which he is timing in order to operate the switch.

Fig. 1. Clock equipped with switch.

Four of these clocks have been in use in the laboratory for two years and no repairs have yet been required. The correction to be subtracted because of the clock coasting after the switch is opened has remained constant to within 0.03 sec for each clock. The clocks have proved to be so satisfactory that we now have fourteen of them in use in our laboratories.

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[1] Am. J. Phys. (Am. Phys. Teacher) **5**, 41 (1937).

The Postprandial Proceedings of the Cavendish Society

Since the appearance of my article under the above title,[1] my attention has been called, by numerous letters, to the following facts. *Ions Mine* (p. 179) was composed mainly by H. A. Wilson, although "J. J.," and possibly others, helped; Durack was therefore the singer and not the author. The song *hν* (p. 245) was written, not by G. Stead, but by G. Shearer, who is now in the x-ray department of the National Physical Laboratory, England. J. A. Crowther informs me that *A Biographical Sketch* (p. 247) was the last song written by A. A. Robb, and was first sung by Crowther at Robb's special request. Finally, "Aethereal Swain" (p. 244) should be *Aethereal Strain;* this typographical error is understandable, for our stenographer is a young woman.

John Satterly

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[1] Am. J. Phys. (Am. Phys. Teacher) **7**, 179, 244 (1939).

Proceedings of the American Association of Physics Teachers

THE COLUMBUS MEETING, DECEMBER 26–29, 1939

THE ninth annual meeting of the American Association of Physics Teachers was held at the Ohio State University, Columbus, Ohio, on December 26–29, 1939. The presiding officers were H. B. Lemon, President of the Association, A. A. Knowlton, Vice President, and R. M. Sutton, President Elect.

In a ceremony held on Wednesday afternoon, December 27, President Lemon and Vice President Knowlton announced the presentation of the 1939 Oersted Medal for Notable Contributions to the Teaching of Physics to *Benjamin Harrison Brown*. An account of the ceremony appears elsewhere in this issue.

A joint dinner with the American Physical Society and Section B of the American Association for the Advancement of Science was held at the Deshler-Wallick Hotel on Friday evening, December 29.

INVITED PAPERS

The following invited papers were heard during one of the sessions:

Research and the College Teacher. Thomas H. Osgood, *University of Toledo, Toledo, Ohio.*

Physics in General Education at the College Level. Lloyd W. Taylor, *Oberlin College, Oberlin, Ohio.*

Preparation and Qualifications of Teachers. Dinsmore Alter, *Griffith Observatory, Los Angeles, Calif.* (Read by T. D. Cope.)

Two invited papers were presented at the joint sessions with Section B, American Association for the Advancement of Science, and the American Physical Society:

The Measurement of Velocity with Atomic Clocks. Herbert E. Ives, *Bell Telephone Laboratories, New York, N. Y.*

Radiofrequency Spectra of Atoms and Molecules. I. I. Rabi, *Columbia University, New York, N. Y.*

CONTRIBUTED PAPERS, WITH ABSTRACTS

Two sessions were devoted to the following contributed papers.

1. (a) A Study of Problem Solving and Achievement in General College Physics. (b) A Study of the Teaching Effectiveness of the Sound Motion Picture, "Light Waves and Their Uses." C. J. Lapp, *University of Iowa, Iowa City, Ia.*—(*a*) During the college years 1937–38 and 1938–39 the writer's premedical classes in general physics used the same textbook and lesson assignments, and the course generally was conducted as nearly the same as was possible. However, in 1937–38 the students were assigned problems, averaging 6 per lesson, were asked to solve them, but were not required to hand in the solutions; whereas, during 1938–39, they were asked to hand in the solutions of these same problems. The same Cooperative Physics Tests were used as semester examinations both years. Two groups from these two years were matched on the basis of their freshman intelligence tests and physics aptitude scores made at the beginning of each college year. When these groups were compared on the basis of achievement in the semester examinations, the one that handed in problems was about 20 percentiles superior.

(*b*) The technic used in evaluating the teaching effectiveness of this sound film was the same as that previously described [Am. Phys. Teacher **7**, 172, 224 (1939)]. As shown by the methods employed, this film is not as effective a teaching instrument as the films previously studied and reported.

2. Encouraging the Use by Students of Review Notes in Examinations. Oswald Blackwood, *University of Pittsburgh, Pittsburgh, Pa.*—Students in reviewing for examinations often waste time and effort in the rote memorization of formulas and definitions. To avoid this practice, we encourage our students to bring to the examination a few, brief notes. The advantages are as follows: (1) In preparing the review notes, the student is encouraged to survey the material, organize his knowledge and clear up points that are imperfectly understood. (2) The morale of the student is improved because he need not waste time in cramming, but can devote it to organizing his knowledge. (3) Honesty is fostered because no student can gain advantage over his fellows by the stealthy use of concealed notes or cribs.

3. (a) All Conversion Factors Are Unity. (b) The Temperature Concept. A. G. Worthing, *University of Pittsburgh, Pittsburgh, Pa.*—An automobile with a speed of 45 mi/hr is brought to rest in 5.0 sec. Therefore, $a=9.0$ mi/(hr sec). To express a in terms of the more satisfactory unit, *ft/sec²*, one may merely multiply the right-hand member by conversion factors, properly expressed, whose values are the simple numeric one. Thus, $a=9.0(\text{mi/hr sec})\times(5280\text{ ft}/1.00\text{ mi})(1.00\text{ hr}/3600\text{ sec})=13.2\text{ ft/sec}^2$. From the student's standpoint this is a far better procedure than just "to multiply by 5280 and divide by 3600;" as many say one should do. In cases that are less simple, where even advanced and graduate students of physics are forced to pause, this procedure has been found to work wonders.

(b) The complete paper appears elsewhere in this issue.

4. Experimenting with Experiments. Louise S. McDowell, *Wellesley College, Wellesley, Mass.*—Last June a questionary was given in the elementary physics course in which students were asked to indicate which experiments were most and least helpful or interesting. From this emerged several facts. In general, the students prefer short experiments or those that have practical applications. They dislike verifying laws already discussed in lectures. Most highly approved were "Series and Parallel Arrangements

of Lamps," "Velocity of Sound," "Spectra and Color," "Machines." The last is not a formal experiment but is the study of a group of machines in common use—such as a bicycle, a block and tackle by which they can lift themselves, an automobile jack—for each of which questions are prepared to test the knowledge of mechanical advantage and efficiency; it might be considered a poor man's substitute for a museum laboratory. Many of the students have never played with electric circuits; setting up a telegraph system with relays helps to overcome that handicap. On the basis of the questionary, more time is being devoted to machines, and the least liked experiments —for example, "The Vernier"—are being shortened and rewritten. The understanding of the theory underlying the experiments is tested in the laboratory by occasional short, objective tests and by individually assigned problems.

5. "Stripped Problems" Tests. Harold K. Schilling, *Union College, Lincoln, Nebr.*—"Stripped problems" are problems almost completely stripped of considerations requring any but purely physical thinking. The computations and the necessity for devoting thought to the creation of appropriate mental images of apparatus or of spatial relationships of objects are reduced to a minimum; while the principles and laws of physics are brought into accentuated relief and the thought processes called for concern themselves mainly with differentiated aspects of the application of those fundamental concepts to particular physical situations. Tests made up of such problems (1) have great diagnostic value; (2) isolate fundamental difficulties in physical thinking from difficulties due to inadequate mathematical equipment; (3) make possible, with a given amount of available time, a more comprehensive evaluation of the student's mastery of a subject; (4) encourage emphasis upon fundamentals in daily study; (5) are exceptionally useful in the study of units; (6) constitute an unusually effective device for the initiation of serious class discussion.

6. The Classification of Motions. Robert S. Shaw, *College of the City of New York, New York, N. Y.*—It is customary to define, name and discuss various types of motions before the laws of motion are studied. The names then assigned are naturally of a kinematical character. In particular, the tendency is to use two names for uniformly accelerated motion. From the dynamical point of view, motions are defined by the special laws of force which cause them; and it would seem natural to name types of motion so as to associate them with the appropriate laws of force. As an introduction to the dynamical viewpoint, it is suggested that, before discussing any *new* type of motion, a brief review be devoted to a dynamical treatment of the simpler types of motion which have already been discussed kinematically. A feature of such a review would be a renaming of these motions. For example, one would speak simply of "motion under a constant force" instead of using a separate name for projectile motion. The purpose of the renaming is to emphasize unity in the treatment of the very diverse types of motions.

7. Physical Quantities and Dimensions. J. Gibson Winans, *University of Wisconsin, Madison, Wis.*—To eliminate confusing definitions, the following axioms may be recognized. A physical quantity (1) is a sense-perceived attribute of matter, (2) is either definable or not definable, (3) cannot be defined in terms of itself, (4) can be defined as a combination of other physical quantities, (5) can be replaced by its definition with no loss of accuracy or generality, (6) is distinct from the act of measuring that physical quantity. Length and time are recognized as indefinable physical quantities. The operational method of definition, which would define length as the comparison of one length with another length, violates axiom (3). If force is defined as *mass*$\times$*acceleration*, much confusion results in definitions of pressure, work, torque, and in description of forces in equilibrium and effects of gravitation, magnetism and electricity. Defining mass as *force/acceleration* produces no such confusion. Force is recognized as an indefinable physical quantity or a directly perceived attribute of matter, while mass is an attribute perceived only through force and acceleration. Dimensions expressed in terms of length, force and time involve the vector character of length and force. Thus, work may be defined as the scalar product and torque as the vector product of length and force. Other physical quantities considered as indefinable are temperature, magnetic pole strength (or magnetic moment) and electric charge.

8. Physics and the Problem of Values. Rogers D. Rusk, *Mount Holyoke College, South Hadley, Mass.*—The general problem of values, once pre-empted by the philosopher, must be recognized by the physicist, and in spite of grave difficulties should be the direct concern of the physics teacher. The laboratory attitude recently extended in one direction to be more critical of the knowing process must now be extended in the opposite direction to be more critical of the results and uses of science. This involves the interpretation of physics in terms of human and social values and raises the problem of the social responsibility of the physicist. The dynamical character of the problem, as well as a wide public interest in its politico-economic aspects, suggests the need for continual and more adequate attempts to interpret the results of physics, to bridge the gap between the more remotely theoretical and the practical phases of life, and to integrate scientific results with social aims. The problem of the relation of such aims to possible scientific procedure is raised. As a preliminary move the desirability is emphasized for a broad survey of the contacts of physics with other fields including brief introductory bibliographies through which easy access for nonspecialists can be attained.

9. Physics for the Masses. C. R. Fountain, *George Peabody College for Teachers, Nashville, Tenn.*—The general public knows little about the nature of physics or its usefulness in everyday life. This is evident from statistics which show that a smaller percentage of high schools are offering physics and that a smaller percentage of students are electing it. This is especially true in the South. Some possible reasons for this situation are: (1) The need for

economy in establishing new schools during the depression has led many administrators to omit from the curriculum subjects that are expensive to house, equip and maintain. (2) Poorly trained teachers do a worse job of teaching physics than those in other subjects, because so much extra time is needed to keep the laboratory and the apparatus in efficient working order. (3) Many school boards demand less specific training for teachers of mathematics and of physics than they do for those of most other subjects. (4) Some states no longer require the mathematics generally considered necessary for the study of physics. (5) Poor teaching makes physics seem difficult and reveals few of its applications and small need for studying it. (6) The general belief among students and parents is that physics is needed only by those preparing for some form of engineering. (7) Many colleges and universities seem to ignore high school physics, all freshmen being put into the same physics classes. (8) Physicists do not advertise their accomplishments. Various means for improving this situation are discussed.

10. An Empiric Approximation of the Vertical Component of the Earth's Magnetic Field for the United States. F. C. Farnham, *Missouri School of Mines and Metallurgy, Rolla, Mo.* (*Introduced by L. E. Woodman.*)—The main features of form and annual rate of secular variation of the vertical component of the earth's magnetic field in the United States can be quite closely represented by a purely empiric relationship. It may be written in the form

$$Z=0.63[0.97992 \sin \phi+0.19937 \cos \phi \cos (\lambda-69)] +k[\sin \phi_0 \sin \phi+\cos \phi_0 \cos \phi \cos (\lambda-\lambda_0)]^n, \quad (1)$$

where Z is the vertical component of the field at a point whose geographical north latitude and west longitude are ϕ and λ, respectively; k, n, ϕ_0 and λ_0 are constants whose values depend upon the epoch for which the field is being computed. For 1935, the values are $k=6835.7$, $n=10.42$, $\phi_0=38.5°$, and $\lambda_0=91.5°$. The agreement between values obtained from Eq. (1) and observed values of the vertical component for 1935 is quite good considering the simplicity of the relationship used. The annual rate of secular variation of the vertical component of the field in the United States can be approximated by assuming certain rates of variation for the constants in Eq. (1). If k is changed at the rate of -52.22 per annum, n at the rate of 0.118 per annum, ϕ_0 at the rate of $-0.15°$ per annum, and λ_0 at the rate of $-0.05°$ per annum, the observed annual rate of change of the vertical component of the field is reasonably well represented.

11. The Training of Arts Majors in Physics for Positions in Industry—A Committee Report. P. I. Wold, *Union College, Schenectady, N. Y.*

12. The Student's Automobile as a Piece of Laboratory Apparatus. J. C. Stearns, *University of Denver, Denver, Colo.*—During the last two years the following experiments have been performed in our course in general physics. Starting with the equations $E=\frac{1}{2}mv^2$, $P=fv$ and $f=ma$, the student develops the formulas for the power and force utilized when the speed of the car is changed from v_1 to v_2 in time t. The speeds v_1 and v_2 are observed on the speedometer, and t is determined with a stopwatch. These data are used to compute the power needed to (1) produce acceleration, (2) overcome over-all friction, (3) overcome engine friction, (4) overcome chassis and road friction, (5) overcome wind friction. The maximum power developed by the brakes in stopping the car is determined. The student plots the power in each of the five cases as a function of the speed and determines how the power and force vary with the speed in each case. The speed for greatest engine economy is determined. Finally, the maximum distance at which the two headlights may be resolved at night is determined; from this, a criterion for passing cars at night is developed.

13. Four Inexpensive Lecture Table Experiments. Richard M. Sutton, *Haverford College, Haverford, Pa.*—(1) *Acceleration in s. h. m.* A liquid manometer with multiplying feature [Sutton, Phys. Rev. **49**, 414A (1936)] is mounted on a long ballistic pendulum with four supporting wires. When the pendulum is set swinging the manometer follows closely in phase the motion of the system and gives visual evidence of acceleration proportional to displacement. The arrangement may be used for the accurate calibration of the accelerometer itself. (2) *Motion of a freely rotating body.* A large wooden disk held in a vertical plane and tossed into the air with a spinning motion rotates about its center of mass, which is clearly marked. The disk is loaded on one side and the mark describes a wobbly motion, whereupon the opposite face of the disk is turned toward the class and the motion is seen to take place about another mark previously concealed. (3) *Torque.* A broomstock 2 cm in diameter and 40 cm long is equipped with a stiff metal rod 100 cm long extending perpendicularly from its mid-point. A weight of 1 kg is hung from the metal rod, while the stick, in a horizontal position, is grasped by the hands. Students may be invited to determine how far from the stick the weight can be placed before it becomes impossible to hold the metal rod in a horizontal plane. By simply turning the stick into a vertical position, the lever-arm of the applied force may be increased and the weight held with ease, even at the end of the rod. (4) *Centripetal force.* Illustrations of centripetal force may be enlivened by the unexpected showing of a rotating doll, the hem of whose skirt is weighted with beads.

14. A Telescope of Very Wide Field of View and Small Diameter-to-Length Ratio. James A. Duncan, *Consolidated Edison Company of New York, Inc., Brooklyn, N. Y.*—A telescope which is employed in taking motion pictures of the interior of large power plant furnaces in action has a number of unusual features. The required real field of view was 50° and the desired photographic speed when used with a 1-in. Ciné lens was $f:12.5$. These had to be attained under conditions that set the minimum length at 18 in. and the maximum diameter at 1.5 in. This is an excellent problem for sophomore students, provided they understand the necessary relations between magnification, field of view, speed and lens diameters.

15. Mechanical Oscillator for Melde's Experiment. P. I. Wold and Frank J. Studer, *Union College, Schenectady, N. Y.*—A new form of rotator head has been designed for setting up transverse waves in a cord by rotating or oscillating the end without giving a twist to the cord. The rotator head is small and is adapted for use with any standard table rotator. The wave set-up is a circularly polarized wave which, for demonstrations, has the advantage of being equally visible from any plane of observation. By a suitable barrier, it can be readily rendered plane polarized. A demonstration was given of the oscillator with one cord and with a plurality of cords.

16. Constructing a Simple Magnetic Lens Electron Microscope. Charles W. Hoffman, *Blair Academy, Blairstown, N. J.*—A 1-m brass tube approximately 6 cm in diameter has a cathode attachment at one end and a fluorescent screen bulb at the other end. The tube is evacuated to about 10^{-4} mm-of-mercury by means of a mercury diffusion pump and liquid air trap. The power supply for the cathode is obtained from the oscilloscope from which the cathode-ray bulb was taken. Around the brass tube are placed two circular iron capsules, each containing a coil of approximately 1000 turns of No. 24 copper wire through which a controlled current passes. The resulting magnetic field focuses the electron beam which then falls on the fluorescent screen to produce a magnified image of the cathode. Objects for investigation may be placed in a special holder attached to the cathode and the corresponding images studied or photographed. The magnification is not high, but the essential principles of electron optics can be demonstrated.

17. (a) Use of Gelatin and Blackboard Chalk Specimens for Elasticity Demonstrations. (b) Simple Demonstration of the Characteristic of an Electron Tube Compared with that of an Ohm's Law Resistance. Eric M. Rogers, *The Putney School, Putney, Vt.*—(a) Blocks and cylinders of jelly are useful both for lantern demonstrations of strain with polarized light and as individual samples for student experiments. Many students grasp the two ways of regarding shear, for example, more easily if they can experiment with samples in their hands. The samples are made by dissolving powdered gelatin in warm water (20 gm of gelatin to 100 ml of water), and pouring into test tubes (for cylindrical samples) and flat dishes (for slabs from which rectangular blocks may be cut). Given a cylinder and blocks of jelly and a stick of chalk, the student tries the following: stretching, compressing, estimating Poisson's ratio, bending a beam, shearing a cube and twisting a cylinder. For the last two cases, ink squares are drawn on the sides (1) with sides parallel to the edges (2) with sides 45° to the edges; the squares (2) shear into rectangles, illustrating the "other way of regarding shear." Finally, the student pushes, pulls, bends and twists the samples until they break. The jelly is found to be stronger for compression than for tension, so when the cylinder is twisted it breaks along a spiral at 45° to the axis. A stick of chalk breaks along the same 45° spiral when twisted. (Concrete behaves similarly—hence the use of reinforcing iron rods.) For demonstration experiments, larger slabs are used in the lantern. Since the jelly becomes doubly refracting under strain, samples can be used with polarized light; they behave like celluloid, except that the strain colors can be produced by a mere touch of the finger.

(b) A simple "mechanical" oscillograph can be arranged using two small, well damped mirror galvanometers G_1 and G_2. A small beam of light is reflected by the mirror of G_1 via two plane mirrors to the mirror of G_2 and thence to a large screen. The two plane mirrors are arranged so that they change the horizontal movement of the light due to G_1 into a vertical movement, while G_2 continues to give a horizontal movement. G_2 arranged as a voltmeter and G_1 as a milliammeter are used to show current-voltage characteristics for (1) a high-resistance coil and (2) an electron tube. When the applied voltage is varied smoothly from, say, $+60$ to -60 v, the spot of light on the screen gives a slanting straight line through the origin for (1) and the usual characteristic for (2). Before showing the characteristics the "axes" are sketched on the screen by using (1) with first G_1 and then G_2 disconnected. Students seem to enjoy seeing this simple oscillograph; the graphs can be made any size.

18. Magnetic Moment. F. W. Warburton, *University of Kentucky, Lexington, Ky.*—Expressing the magnetic moment of a magnet in *ampere meter*^{2} facilitates applying the language of currents directly to magnets. It is assumed that the alinement of atoms in the surrounding medium and also in the magnets themselves changes in such a way as to provide the energy of magnetizing the medium, $U=\int\int HdIdv$, at the expense of the magnets. This accounts for the energy needed and for the reduced force on two coaxial magnets when immersed in a magnetic medium or when separated by a thin sheet of soft iron. In mks units the intensity of magnetization I then is similar to H. H is physically the concentration of currents together in a coil expressed in ampere turns per meter, while I becomes the concentration of amperian currents by the alinement of atoms expressed in amperian amperes per meter; hence the suggestion that H be termed *intensity of source*, releasing the name *field* for B as equal to the force on unit length of unit current. In place of the name "magnetomotive force" for "work per pole," the closed line integral, $\oint Hdl$, expressed in ampere turns, may have the concordant name *magnetomotive source*.

19. A Demonstration of Center of Gravity, Moment of Inertia and the Period of a Compound Pendulum. W. B. Pietenpol, *University of Colorado, Boulder, Colo.*—Two meter sticks of equal weight and mass distribution are supported at their ends so that they may swing about a horizontal axis. Each stick supports a pair of equal weights, the positions of which may be adjusted. When the sticks are held in a horizontal plane by means of a cord over a pulley, it is shown that the moments are the same, provided the centers of gravity of the two sticks with their weights are in the same relative positions; this demonstrates that the weight of a body can be correctly represented by a single force acting at the center of gravity. When the

sticks are considered as pendulums, it is shown that, for the same centers of gravity and the same moments of inertia, the periods of vibration of the two are the same. When, however, the positions of the weights on one stick are changed, retaining the position of the center of gravity and consequently the torque acting, the change in the moment of inertia causes a pronounced difference in the periods of vibration of the two pendulums. By the coincidence method the relative moments of inertia of the two pendulums can be approximately obtained.

20. Three Pieces of Equipment for the Museum or Demonstration Laboratory. J. G. Black, *Morehead Teachers College, Morehead, Ky.*—(1) A Foucault pendulum has a soft steel core which is vertical when the pendulum is at rest. Beneath this core is an electromagnet which pulls the displaced pendulum to the center. When the pendulum passes through the center the circuit is opened by a relay operated by a mercury contact or by the induced current from a small coil surrounding the end of the magnet. The circuit does not close again until after the pendulum has reached its maximum displacement. Thus, continuous operation with any desired amplitude may be had with no mechanism visible, except possibly the mercury pool at the center.

(2) A Wheatstone bridge has two arms consisting of equal circular resistors, one of which is fixed while the other is variable and has a dial graduated in percentage from 1 to 100. The third resistor is variable in steps of 10 from 1 to 10^6. The fourth resistor is the unknown. The reading of the dial is a direct measure of the unknown because a setting of 43 on the dial indicates 43/100 of the third resistance.

(3) Some laboratories use a device by which the student measures his own horsepower. It consists usually of a mounted pulley which is provided with a crank and which has a Prony brake strap passing over it, weighted on each end. As one faces the machine, the crank is turned clockwise and the heavier weight is on the left. To this standard design the present paper adds two features which make the machine better adapted to use in a museum or demonstration laboratory. (a) A Prony brake with a variable coefficient of friction is used. The left side of the brake strap is lined with a flexible metal strip, or provided with rollers. As the speed increases the metal climbs upward on the pulley, thus permitting an increase in speed without changing the weight. With the radius and force on the pulley constant, the speed becomes a direct measure of the horsepower. (b) The speed is obtained by a speedometer calibrated to read directly in horsepower. The machine may be used by the elementary laboratory student who takes all measurements or it may be used in the museum to read directly without measurements.

21. Classroom Demonstration of the Nature of the Charge on the Electron. A. D. Hummel, *Eastern Kentucky State Teachers College, Richmond, Ky.*—In Perrin's cathode-ray vacuum tube, cathode particles are projected through an opening in the anode and deflected by an external magnet into a Faraday cylinder. The directions accompanying this demonstration tube simply require that the collecting electrode be connected to an electroscope. The author has found that these directions lead to the conclusion that cathode particles are positive. The explanation proposed is that the positive ions diffuse into the cylinder and mask the effect of the electrons. To prevent this, a suitable retarding potential is applied through a variable condenser in parallel with the electroscope; then the electroscope will show no charge unless the cathode rays are deflected into the cylinder. The charge collected may then be shown to be negative. Care must be taken to avoid induced charges. The author used a two-plate condenser whose capacitance was varied by changing the distance between plates. After collecting the cathode particles, the upper plate was lifted to increase the deflection of the electroscope before testing its charge.

22. The Structure of a Liquid. C. D. Thomas, *Missouri School of Mines and Metallurgy, Rolla, Mo.* and Newell S. Gingrich, *University of Missouri, Columbia, Mo.*—Within recent years, work on the diffraction of x-rays by liquids has supplied considerable information concerning the structure of liquids. The presentation to undergraduate students of the structure of crystals and of the structure of liquids can be made at the same time to emphasize the points of similarity and of difference between the solid and liquid states. The atomic distribution curve expresses the "structure" of the liquid; and it is interesting to compare the distribution curve for a liquid element with the corresponding curve for the same element considered as an ideal crystal. Prins and Wall have given approximate mathematical expressions for the atomic distribution curves which can be used to convert an ideal crystal distribution curve to that for a liquid, and this can be compared with the experimentally determined distribution curve for the liquid. Comparisons of this sort have been made for some of the liquid elements to illustrate the feasibility of presenting the concept of atomic distribution function to undergraduate students.

23. An Experiment With Written Recitations. R. B. Abbott and H. H. Remmers, *Purdue University, Lafayette, Ind.*—Written recitations were tried with a class of 750 sophomore engineering students taking general physics. The course consisted of 1 demonstration lecture, 1 laboratory experiment and 3 recitations per week for 1 semester. Multiple-choice questions were used at each recitation. The papers were graded and returned to the students for discussion at the beginning of the next recitation. Comparative results were obtained by giving the *Cooperative Physics Tests* as a final examination. The results were better than had been expected. A poll of the students at the end of the semester showed over 90 percent in favor of the methods used.

24. Apparatus for a Laboratory Experiment Leading to the Postulation of Newton's Laws of Motion. Nicholas M. Smith, Jr., *University of Chicago, Chicago, Ill.*—It has been pointed out by many writers, notably Mach and Eddington,

that the usual method of presenting the concepts of mass and of force (by giving simply the laws of motion as stated by Newton) is logically incomplete. In a final analysis this method assumes that the student already possesses an intuitive concept of these important quantities. Of all the experiments that may be devised to show the postulation of Newton's laws, that by Mach can be best taught to beginners, in that it shows clearly how they obtain their intuitive ideas of mass and of force. An apparatus to perform this experiment has been devised and was demonstrated. The experiment consists essentially of causing a "standard" object (a car on a track) to interact by means of weights or springs with an "unknown" object (car) A. The ratio of the acceleration of the standard, a_{SA}, to that of the other car, a_{AS}, with negative sign (because they are oppositely directed), is obtained by measuring the ratio of distances the cars S and A travel in equal times. This ratio is shown to be a constant independent of the degree of interaction and is subsequently defined as the mass m_A of A. By accelerating the standard car S against a third car B it is shown *experimentally* that $(-a_{SA}/a_{AS})/(-a_{SB}/a_{BS}) \equiv m_A/m_B = -a_{BA}/a_{AB}$. By writing this as $m_A a_{AB} = -m_B a_{BA}$ and defining the force of B on A as $m_A a_{AB}$, and the force of A on B as $m_B a_{BA}$, one obtains immediately the three laws of Newton. This method of presentation to beginners has the important advantages that (1) it is logical, (2) shows definitely the postulatory character of the laws of motion, (3) shows clearly that by a force we mean the effect of all other objects *on* a given object (that there is no such thing as an isolated force) and (4) by having a student estimate the mass of an object, say, a book, demonstrates that in everyday life he performs essentially this experiment of Mach's with himself as the standard object and thus arrives at his intuitive concepts of mass and of force.

Attendance

The registration of those in attendance lists 130 members of the Association and 49 nonmembers. Members who registered were:

R. B. Abbott, Purdue University; W. M. Baker, University of Detroit; P. D. Bales, Howard College; I. A. Balinkin, University of Cincinnati; H. A. Barton, American Institute of Physics; V. P. Barton, Goucher College; P. Bender, Goshen College; C. E. Bennett, University of Maine; F. L. Berger, Ohio Northern University; J. G. Black, Morehead State Teachers College; F. C. Blake, Ohio State University; L. I. Bockstahler, Northwestern University; A. B. Cardwell, Kansas State College; T. J. Carroll, College of New Rochelle; A. W. Coven, Kent State University; C. W. Chapman, Michigan State College; F. F. Cleveland, Armour Institute of Technology; T. D. Cope, University of Pennsylvania; S. W. Cram, Kansas State Teachers College; R. C. Ditto, Alma College; H. L. Dodge, University of Oklahoma; J. A. Duncan, Consolidated Edison Company of New York; C. H. Dwight, University of Cincinnati; R. L. Edwards, Miami University; D. S. Elliott, Tulane University; J. E. Evans, Rio Grande College; C. R. Fountain, George Peabody College for Teachers; O. R. Fonts, University of Akron; Sister Mary Charlotte Fowler, Nazareth College; G. C. Fromm, Capital University; H. Q. Fuller, Albion College; W. H. Gran, Western College; M. B. Grandy, University of Dayton; G. E. Grantham, Cornell University; R. R. Hancox, Olivet College; R. E. Harris, Lake Forest College; R. J. Havighurst, General Education Board; J. J. Heilemann, American Philosophical Society, Philadelphia; L. B. Heilprin, Northeastern University; M. E. High, Minnesota State Teachers College; W. L. Hole, Elmhurst College; R. M. Holmes, University of Vermont; F. F. Householder, University of Akron; R. H. Howe, Denison University; A. G. Hoyem, Augustana College; C. W. Huffman, Blair Academy; A. D. Hummel, Eastern Kentucky State Teachers College; E. Hutchisson, University of Pittsburgh; J. M. Kelley, Loyola High School, Baltimore; W. Kinkaid, Mississippi State College; P. E. Klopsteg, Central Scientific Company; H. C. Knauss, Johns Hopkins University; H. P. Knauss, Ohio State University; A. A. Knowlton, Reed College; E. J. Kolkmeyer, Canisius College; C. J. Lapp, University of Iowa; K. Lark-Horovitz, Purdue University; K. G. Larson, Augustana College; G. Q. Lefler, Kent State University; H. B. Lemon, University of Chicago; H. W. Le Sourd, Milton Academy; Sister Grace Marie, College of Chestnut Hill; M. J. Martin, University of Wisconsin; P. E. Martin, Muskingum College; O. E. McClure, Ohio University; F. C. McDonald, Southern Methodist University; Louise S. McDowell, Wellesley College; J. H. McMillen, Kansas State College; J. A. Mescher, De Sales College; Helen A. Messenger, Hunter College; W. H. Michener, Carnegie Institute of Technology; C. W. Miller, Michigan State College; J. G. Moorhead, Westminster College; C. C. Murdock, Cornell University; W. Noll, Berea College; T. H. Osgood, University of Toledo; H. N. Otis, Hunter College; G. E. Owen, Antioch College; F. Palmer, Haverford College; R. A. Patterson, Rensselaer Polytechnic Institute; J. R. Patty, Arkansas State College; G. T. Pelsor, University of Oklahoma; C. J. Pietenpol, Washington and Jefferson College; W. B. Pietenpol, University of Colorado; W. J. Poppy, Fenn College; R. A. Porter, Syracuse University; C. W. Prine, Carnegie Institute of Technology; M. J. Pryor, Houghton College; W. R. Pyle, Wilmington College; O. L. Railsback, Eastern Illinois State Teachers College; E. M. Rogers, The Putney School; D. Roller, Hunter College; Y. K. Roots, Findlay College; H. H. Roseberry, Ohio University; G. Rosengarten, Philadelphia College of Pharmacy and Science; R. D. Rush, Mount Holyoke College; H. K. Schilling, Union College; J. M. Schmidt, Hofstra College; G. K. Schoepfle, A and M College of Texas; C. C. Schubert, Riley High School, South Bend, Ind.; R. S. Shaw, College of the City of New York; W. E. Singer, Bowling Green State University; F. G. Slack, Vanderbilt University; H. L. Smith, Michigan State Normal College; L. E. Smith, Denison University; A. D. Sprague, Elon College; R. J. Stephenson, University of Chicago; G. W. Stewart, University of Iowa; W. W. Stifler, Amherst College; R. M. Sutton, Haverford College; V. F. Swaim, Bradley Polytechnic Institute; J. A. Swindler, Westminster College; L. W. Taylor, Oberlin College; M. D. Test, Lawrence Institute of Technology; F. G. Tucker, Oberlin College; E. C. Unnewehr, Baldwin Wallace College; F. L. Verwiebe, Eastern Illinois State Teachers College; L. Vollmayer, John Carroll University; C. N. Wall, North Central College; F. W. Warburton, University of Kentucky; Dorothy W. Weeks, Wilson College; W. R. Westhafer, College of Wooster; N. E. Wheeler, Colby College; M. W. White, Pennsylvania State College; J. G. Winans, University of Wisconsin; W. C. Wineland, University of Kentucky; P. I. Wold, Union College; R. M. Woods, Northwestern University; A. G. Worthing, University of Pittsburgh; Charlotte Zimmerschied, Southern Illinois Normal University; Sister Mary Therese, Mundelein College.

Annual Report of the Treasurer

Balance brought forward from Dec. 15, 1938		$1774.26
CASH RECEIVED		
Dues received[1] for 1939	$4060.00	
Dues received for 1938	15.00	
Dues received for 1940	115.00	
Grant	1500.00	
Royalties, *Demonstration Experiments in Physics*	610.83	
Donations	36.35	
Total Cash Received	$6337.18	
Total deposited from 12/15/38 to 12/15/39		6337.18
Total cash available		$8111.44

DISBURSEMENTS

Postage and supplies	$ 206.42	
Printing	146.89	
Secretary's office expense	254.12	
Stenographic service, Editor's office.	554.40	
Editor's traveling expense	44.84	
Payments to American Institute of Physics	2328.02	
Discount charge and call by bank	0.63	
Journal survey articles	100.00	
Money advanced on *Demonstration Experiments in Physics*	610.83	
Expense of Stanford Meeting	9.77	
Total Disbursed		4255.92
Balance on hand[2] Dec. 15, 1939		$3855.52

PAUL E. KLOPSTEG, *Treasurer*

I have audited the books of account and records of Dr. P. E. Klopsteg, Treasurer of the American Association of Physics Teachers, for the year ended December 15, 1939, and hereby certify that the foregoing statement of receipts and disbursements correctly reflects the information contained in the books of account. Receipts during the year were satisfactorily reconciled with deposits as shown on the bank statements, and all disbursements have been satisfactorily supported by vouchers or other documentary evidence.

WILLIAM J. LUBY, C.P.A.

Chicago, Illinois,
December 21, 1939.

[1] On December 15, 1939 there were 848 members in good standing.
[2] A balance of approximately $1800 is due the American Institute of Physics for the publication of the journal during 1939.

Report of the Secretary

The Executive Committee of the American Association of Physics Teachers met three times during the Columbus meeting. Members present were T. D. Cope, D. S. Elliott, R. E. Harris, A. D. Hummel, A. A. Knowlton, P. E. Klopsteg, C. J. Lapp, K. Lark-Horovitz, H. B. Lemon, Louise McDowell, W. H. Michener, W. B. Pietenpol, F. Palmer, D. Roller, R. M. Sutton, A. G. Worthing. Those present by invitation were F. C. Blake, H. P. Knauss, F. G. Slack and G. W. Warner.

A committee consisting of H. L. Dodge, W. E. Forsythe and P. E. Klopsteg was appointed to confer with other groups and societies known to be interested in instituting a memorial for the late Professor F. K. Richtmyer.

It was voted that the financial support which the Association gives the American Institute of Physics be increased from 15 to 20 percent, thus extending for another year the increased rate first authorized in June, 1938. As representatives of the Association on the governing board of the American Institute of Physics, G. R. Harrison was nominated to serve from 1940 to 1943 and F. Palmer was nominated to fill the unexpired term of F. K. Richtmyer. Approval was given to a proposed joint meeting of the founder societies of the American Institute of Physics in 1941.

The secretary reported that: the Association has become affiliated with the Pacific Division of the American Association for the Advancement of Science; that complete and satisfactory reports had been received from the seven local chapters of the Association. He further reported that ballots for the proposed amendments to the constitution, authorized at the last annual meeting, had been mailed to members of the Association on June 1, 1939 and that the results were as follows: 285 in favor, and 13 not in favor of adding Art. III (2) (*c*) to the constitution; 264 in favor, and 34 not in favor of other proposed changes serving to create junior memberships.

Actions taken concerning the journal included: the change of name from *The American Physics Teacher* to *American Journal of Physics;* publication of a supplement early in 1940 containing a list of members, the constitution and by-laws, and other information concerning the Association; the establishment of a committee on necrology; appointment of W. H. Michener, J. R. Nielsen, P. I. Wold and M. W. Zemansky as associate editors for the period 1940–1942.

Reports were received from nine special committees of the Association that served during 1939. The following committees and chairmen were requested to continue work during 1940: Physics in relation to medical education, W. E. Chamberlain; Tests and testing, C. J. Lapp; Training of physicists for industry, P. I. Wold; Terminology, symbols and abbreviations, D. Roller and H. K. Hughes. Committees discharged with a vote of appreciation for their work were: Opportunities for greater service, F. G. Slack; Manual of demonstration experiments, R. M. Sutton; *Science Leaflet*, L. W. Taylor; Improving interrelations of physics and physics teachers in colleges and secondary schools, A. W. Smith; nominating committee for 1939, F. G. Slack. W. H. Michener, D. S. Elliott and C. J. Lapp were appointed members of the nominating committee for 1940.

It was voted to apply for constituent membership in the American Council on Education. A committee consisting of K. Lark-Horovitz, *chairman*, G. W. Warner, H. W. Le Sourd and R. J. Stephenson was appointed to explore the possibilities for concerted action with interested groups of mathematicians and chemists in correcting unsatisfactory conditions in elementary and secondary education which affect the preparation of students for the study of physical science in college. The secretary was instructed to transmit to the Physics Section of the Central Association of Science and Mathematics Teachers the offer of possible help with specific problems of physics teaching that might be suggested by the Section. It was also voted to cooperate in all ways possible with the Science Masters Association of Great Britain.

The invitation of the American Association for the Advancement of Science to participate in its 1940 summer meeting, at Seattle, Washington, was accepted.

The Annual Business Meeting. The annual business meeting, held in the Chemistry Building of the Ohio State University, was called to order by President Lemon at 3:45 P.M., December 27.

On behalf of the executive committee, P. E. Klopsteg read a memorial to the late Past-President F. K. Richtmyer. A resolution that the memorial be made a part of the permanent records of the Association, and that copies be transmitted to Mrs. Richtmyer and to the Department of Physics of Cornell University was carried by a rising vote followed by a period of silence.

W. H. Michener reported for the tellers that the result of the election of officers for 1940 was as follows:

President: R. M. SUTTON.
Vice President: A. G. WORTHING.
Members of the Executive Committee: LOUISE S. MCDOWELL, L. W. TAYLOR.

Upon motion without dissent, E. C. Watson was appointed a member of the Executive Committee to complete the unexpired term of R. M. Sutton.

The treasurer, P. E. Klopsteg, presented his annual report. The secretary reviewed the actions of the executive committee. F. C. Blake explained the advantages to the Association of seeking constituent membership in the American Council on Education.

By unanimous vote, the local committee of the Association was thanked for its services in making arrangements for the meeting.

THOMAS D. COPE, *Secretary*

Floyd Karker Richtmyer

1881–1939

IN the sudden passing, on November 7, 1939, of Professor Floyd K. Richtmyer, the American Association of Physics Teachers has lost a highly esteemed member, a good friend, a distinguished past president, an enthusiastic supporter of the objectives of the Association. Well known throughout the scientific world and to a large section of the world of industry, his renown brought distinction to our society. At the time of his death he was one of our representatives on the Governing Board of the American Institute of Physics, as well as a member of the Executive Committee of the Governing Board.

Professor Richtmyer's memberships in the learned societies and his honors were numerous. The work he was called upon to do for those societies testifies to his unique ability to carry out such work exceedingly well and willingly. Although the busiest of men, he was never too busy to accept additional responsibility when that which he was asked to undertake accorded with his beliefs; nor was he ever too busy for a friendly greeting to any one of the great number of his acquaintances.

When the American Association of Physics Teachers was organized in 1930, at the Cleveland meeting of the American Association for the Advancement of Science, Professor Richtmyer was present at the preliminary discussion and later at the organization meeting. To him went membership card No. 1 in the Association. Thus he was our first member. Throughout the nine years of his connection with the Association he remained its first member not only in point of numerical sequence, but also in his energetic and well-directed activity towards achieving valuable results in its undertakings. The period of his administration as president, 1937 and 1938, was the period in which the Association arrived at the point in its development when it could truly be said that we were an established society qualified to undertake serious business, and assured both by membership and financial resources of a permanent place among American learned societies. He had helped to build the membership, in work that had been so well started by his predecessors in office. Single-handed he obtained the five-year annual grant to assist the work of the Association. He succeeded in showing those in the position to make such a grant that we had attained stability and that we could be trusted to use wisely and to good purpose the funds which might be allotted to us. We shall not fail to prove to the grantors that he was right in his representations.

Mr. President, I am deeply moved by the privilege you have accorded me of paying this tribute to one who was my friend for more than twenty years. I ask your permission to extend that privilege to all of our members. Permit me, therefore, to move that the members of the American Association of Physics Teachers, assembled in annual meeting, express their sense of great loss in the death of Professor Floyd Karker Richtmyer; that this memorial and resolution be made part of the permanent records of the Association by inscribing them in the minutes of this meeting; and that the secretary be instructed to send a copy of these proceedings to Mrs. Richtmyer and to the Department of Physics, Cornell University, together with an expression of our very sincere sympathy in their greater loss.

PAUL E. KLOPSTEG
For the Executive Committee

Jonas Bernard Nathanson

1889–1939

ON November 25, 1939, death came without warning to Jonas Bernard Nathanson of the Carnegie Institute of Technology. He collapsed while preparing to attend the theater with his family, and death came shortly after the arrival of the physician. Thus an able teacher, an accomplished scholar, an earnest seeker after truth, a good and loyal friend is gone from our midst.

Jonas Bernard Nathanson was born on September 5, 1889, in Vilna, Lithuania. He came to this country when quite young and received his early education in the schools of Toledo, Ohio. He attended Ohio State University, where he received his A.B. degree in 1912. His graduate work was done at the University of Illinois, where he received the A.M. degree in 1913 and the Ph.D. degree

in 1916. During the latter three years he also served as assistant in physics. From 1916 until his death he served at the Carnegie Institute of Technology as Instructor, Assistant Professor and Associate Professor of Physics.

While at the University of Illinois he became interested in the optical properties of the alkali metals. This interest continued during the remainder of his career, and he contributed many papers in this field. He also did research on the ratio of the charge to the mass of the electron, on interference in metallic films and on optical dispersion of metals. He was invited to speak many times on these subjects before scientific societies.

He was a member of Phi Beta Kappa, Sigma Xi, Phi Lambda Upsilon, American Association for the Advancement of Science, American Physical Society, Optical Society of America, Physical Society of Pittsburgh (President, 1929) and the American Association of Physics Teachers.

Doctor Nathanson will be remembered not only for his contributions to research, but also, and perhaps primarily, for his success as a teacher. Any course which he gave was taught in a thorough and scholarly manner. His presentation of a subject was exceptionally clear and well-ordered. He expected conscientious and scholarly work by his students, and he himself set a superb example. His lectures were masterly both in the selection and presentation of the subject matter and in the demonstrations. Much time was spent in designing and assembling apparatus for these demonstrations. He will long be remembered for his enthusiasm, his perseverance, his sincerity and his modesty.

Chas. W. Prine

Junior Membership in the Association

IN accordance with a recent amendment of the constitution of the American Association of Physics Teachers there may be elected as a *junior member*, any college or university student who has a major interest in physics and whose previous work therein is equivalent to at least two one-year courses of collegiate grade. Junior members, for whom the annual dues are $2.50, receive the journal and have all privileges of the association except voting and holding office. An individual may be classed as a junior member only until the end of the third year after receipt of his bachelor's degree at which time his connection with the association shall cease unless on application he shall be elected to *membership*.

Change in Name of the Journal

THE change in name of this journal to *American Journal of Physics*, effective with the present issue, was authorized by the Executive Committee of the American Association of Physics Teachers during the ninth annual meeting at the Ohio State University. The new name is one of several considered for adoption at the time when the journal was founded, in 1933. The desirability of changing the name has repeatedly been the subject of informal discussion, and was considered by the Executive Committee as a whole during the eighth annual meeting, in 1938.

Experiences over a period of seven years in editing the journal and in dealing with authors and other physicists, and with administrative officers in the institutions of authors, have made it evident that the former name, *The American Physics Teacher*, failed to symbolize the conceptions of physics teaching, the academic grade and the standards of quality that the association and journal seek to foster, and that regular readers have come to expect of the journal. Thus the change in name implies, not an abandonment, but a continuation and improvement of editorial policies and objectives as they are exemplified in the issues of the past several years.

Since its inception, the journal has sought to foster broad and comprehensive conceptions of the place of physics in our modern culture, and to provide material for improving all aspects of physics instruction. Interpreted in a wide cultural sense, improvement in instruction involves not only the development of better teaching methods and facilities but the encouragement of a wide range of interests and activities on the part of physicists who teach. The journal should provide material that will encourage and assist teachers and students in all types of institutions to keep abreast of the state of the science, and to engage continually in creative work of one kind or another. It should seek to promote a better understanding of the objectives and of the difficulties inherent in the various applied and borderland fields of physics, and an increased realization that the philosophic, historical and socio-economic aspects of physics are best regarded as integral parts of the science, the science itself in turn being an integral part of our whole culture.

Actually and potentially, teaching affords the largest field for the employment of physicists, the means by which practically every physicist receives his basic training and the most effective method available for the general dissemination of physical knowledge. The purpose of the *American Journal of Physics* is to assist in every way possible in the improvement and growth of this important and indispensable activity of the profession.—The Editor.

RECENT PUBLICATIONS AND TEACHING AIDS

Advanced Textbooks and References

Recent Advances in Surface Chemistry and Chemical Physics. Edited by Forest Ray Moulton. 133 p. 19×26 cm. *Science Press*, $2.50. The two unified series of invited papers which appear in this volume were originally presented in symposiums organized by the Section on Chemistry of the American Chemical Society. The one series, on the applications of surface chemistry to biology, was prepared by I. Langmuir, W. D. Harkins, L. H. Germer, H. Sobotka and G. H. A. Clowes. The other series on recent advances in chemical physics, deals with fundamental questions concerning the structure and properties of elements and compounds, the contributors being H. C. Urey, J. Y. Beach, C. P. Smyth, M. Randall, H. Sponer, J. H. Hibben and A. Sherman. Each paper is documented.

Electricity and Magnetism. John B. Whitehead, Professor of Electrical Engineering, The Johns Hopkins University. 233 p., 96 fig., 21×14 cm. *McGraw-Hill*, $3. This compact and clearly written introduction to the theory of electricity and magnetism is intended for those upper-division students of electrical engineering and physics who must acquire the essentials of the theory in a relatively brief course. The treatment is nonvectorial. Although the simple electron picture of matter is introduced to some extent, the emphasis throughout the text is on the mechanical character of the concepts upon which electric and magnetic measurements and utilizations are based. The first six chapters deal with electrostatics; the remaining nine, with magnetostatics, electrodynamics, electromagnetism, units, electromagnetic induction, transient circuit conditions, alternating currents, methods of computation for a.c. circuits, and conduction in gases.

History of Science

A Short History of the Steam Engine. H. W. Dickinson. 270 p., 10 plates, 3 tables, 78 fig., 15×24 cm. *Cambridge Univ. Press* and *Macmillan*, $3.50. The steam engine is not only more important today than ever before in the world's economy but its history is gaining in significance and interest for the physicist with the increasing realization that the history of the sciences should not be separated from the histories of their applications, of inventions and of our culture as a whole. Intimately related to the history of physics are reciprocating engines, steam turbines and boilers; and it is to their developments that the present book is restricted. The subject matter is technical and authentic, and the approach is socio-economic and humanistic, showing the vast amount of thought and experimentation that has gone into the development of steam engines. The style is so unusually clear and interesting that it should appeal to the general reader as well as to the serious student. The diagrams and plates are excellent. The author is well known as a biographer of Watt, Boulton, Trevithick and Fulton, and as the collaborator with Rhys Jenkins on *James Watt and the Steam Engine*, a memorial volume prepared for the Watt centenary commemoration at Birmingham, England, in 1919.

The Concepts of the Calculus. Carl B. Boyer, Rutgers University. 353 p., 22 fig., 15×23 cm. *Columbia Univ. Press*, $3.75. Because of the readiness with which the definitions of the derivative and the integral, and the operations involving them, can now be mastered, and because of the character of most textbooks on elementary calculus, the student of today is likely to acquire little appreciation of the evolutionary character of these concepts, or of difficulties encountered in developing them from their incipiency in antiquity to their final elaboration as mathematical abstractions. The present book should therefore be of value, not only to workers in scientific history, but to any serious reader who has had elementary calculus and who wishes to gain a better idea of the historical development of ideas; and this the more so because the book refrains from being comprehensive and elaborate in detail, affording instead a critical tracing of basic concepts. For those students who wish to go into the history more thoroughly, a bibliography is provided, the extensiveness of which testifies to the voluminous literature available on the origin and subject matter of the field. Funds to assist in the publication of this volume were contributed by the American Council of Learned Societies.

A Short History of Science. W. T. Sedgwick, H. W. Tyler and R. P. Bigelow, Massachusetts Institute of Technology. 529 p., 1 plate, 61 illustrations, 14×21 cm. *Macmillan*, $3.75. Like Sedgwick and Tyler's *A Short History of Science* (1917), of which it is in part a revision, this work provides a broad, general perspective of the evolution of the physical and biological sciences, including mathematics. The space devoted to mathematical science has been reduced in the present edition by the omission of relatively technical material and quotations. No attempt has been made to bring the history up to date in such matters as recent physics and the advances in chemistry and biology characteristic of the present century, the authors' contention being that these advances have an abundant, available literature, and that they are probably still too close to our generation for a just historical perspective. However, the attempt is made to trace briefly the foundations upon which recent, as well as earlier, advances are based. Increased emphasis is also placed on the evolution of scientific methods. The present edition

was prepared by the two junior authors, a mathematician and a zoologist, respectively, with the assistance of scientists representing various other fields.

Photography

The Photographic Process. Julian Ellis Mack, Assistant Professor of Physics, University of Wisconsin, and Miles J. Martin, Professor of Physics, Milwaukee Extension Center, University of Wisconsin. 586 p., 15 pictorial studies, 253 fig., 7 tables, 18½×25 cm. *McGraw-Hill*, $5. This excellent textbook should do much to convince physicists of the propriety of offering photography under the auspices of the college physics department. Although the authors have not in any sense neglected the chemical and esthetic aspects of the subject, a perusal of their book will show the close connection between the photographic process and physical science. Martin has described elsewhere [Am. Phys. Teacher **7**, 116 (1939)] how such a course might be organized. The present textbook may be used with undergraduates who have had no training in physics or, by inclusion of the more advanced material printed in small type, with students who have had a year or more of college physics. The general reader will find much information of a practical nature not ordinarily included in the conventional popular treatise. The authors are to be especially commended for the excellence of the photographs they have employed as examples of both photographic technic and pictorial composition; in too many instances the quality of the photographs found in books on photography is not such as to inspire the confidence of the reader in the author's photographic ability. The chapters on the basic photographic process are complemented by chapters on the history of photography, natural-color photography, scientific and technologic photography, photomechanical reproduction and pictorial photography. Also included are a comprehensive bibliography, appendixes on mathematics and chemistry, a formulary and a manual of 31 laboratory experiments. We believe that an abridged edition of this book, containing the entire chapters on the basic photographic process and the appendixes, formulary and experiments reduced in scope, would be of great value as a textbook for a short course where the price of the complete book might be prohibitive.—W.W.

Mathematical and Physical Tables

Duodecimal Arithmetic. George S. Terry. 301 p., 1 fig., 23×29 cm. *Longmans, Green*, $7.50. The author presents, in a sane and logical manner, the arguments for a number system based on twelve instead of ten, notably the point that twelve has six factors as opposed to four factors for ten. In the decimal system, digits from right to left indicate units, tens, ten-squareds, etc. In the duodecimal system, the digits indicate units, twelves, twelve-squareds, etc., and the "decimal point" indicates twelfths instead of tenths. The book contains many trigonometric, logarithmic and other tables based on the duodecimal system.—D. H. D. R.

Vapor Charts and Special Tables for Turbine Calculations. Frank O. Ellenwood and Charles O. Mackey, Professors of Heat-Power Engineering, Cornell University. 47 p., 17 charts, 7 tables, 28×21 cm. *Wiley*, $2.50. The charts contained in this book show the thermodynamic properties of steam, water, ammonia, freon, and mixtures of air and water vapor; the tables give barometric corrections, jet velocities from an ideal nozzle and squares of number. The steam charts, in which specific enthalpy (B.t.u.$\cdot$lb^{-1}) is plotted as a function of specific volume, cover the large specific volume range 0.05 to 3500 ft^3 lb^{-1}. As in the case of the senior author's earlier and less comprehensive *Steam Charts* (1914), the present charts are divided into parts of book-page size; this form is more durable, and more convenient for quick and accurate reference, than charts printed on large, folded sheets.

Popular Books

The World Around Us. Paul Karlson. 303 p., 156 fig., 4 tables, 8 plates, 15×23 cm. *Simon and Schuster*, $3. A selection of the Scientific Book Club, this "modern guide to physics," presents for the nonscientific reader a popular but scientifically accurate and up-to-date description of the world of physical science. The initial sections on matter, electricity and light waves describe the most important and fundamental phenomena, experiments and theories of classical physics and lead logically to the later sections on the more difficult topics of modern physics—relativity, quantum theory, the newer developments in atomic and nuclear physics, and the wave theory of matter. The skilful use of similes, many of them humorous but to the point, the inclusion of human interest anecdotes in connection with many of the great names in physics, the amusing pen-and-ink sketches and the conversational style of the author, a trained journalist as well as a scientist, all add to the popular interest of the book.—H.N.O.

Pamphlets and Catalogs

Electrical Measuring Instruments for Research, Teaching and Testing. 66 p. *Leeds & Northrup Co.* (4934 Stenton Ave., Philadelphia), gratis. A condensed catalog of the entire Leeds & Northrup line of instruments for research, teaching and testing.

Broadcast Receivers and Phonographs for Classroom Use. 95 p. *Committee on Scientific Aids to Learning, National Research Council* (41 E. 42nd St., New York), gratis to teachers. A general discussion of the factors which should be considered in the selection of broadcast receivers and phonographs for classroom use.

Motion Picture Films

Displacement Method of Finding Density of an Irregular Object. 16-mm silent film, 8 min. *Harmon Foundation* (140 Nassau St., New York), rental. Shows the complete experiment and the necessary calculations.

DIGEST OF PERIODICAL LITERATURE

An adjustable curve. H. H. Macey; *J. Sci. Instr.* **16**, 90, Mar., 1939. A 10-in. hacksaw blade with teeth removed, bowed by a screw as shown in Fig. 1, serves for drawing

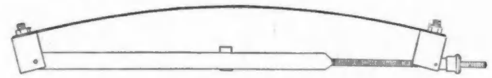

Fig. 1. An adjustable curve.

smooth lines of reasonable length and small curvature. A piece of $\frac{1}{2}$-in. brass, $\frac{3}{4}\times\frac{5}{8}$ in., is fastened to each end. A strip of light brass, 7 in. long, is pivoted in a slot at one end while a threaded rod soldered to this strip passes through a collar pivoted at the other end, a terminal head providing the adjustment. The curve, after adjustment, is held firmly with the left hand by means of a small piece of brass soldered to the under side of the brass strip. The blade, which should not touch the paper, possesses some lateral give which may be an advantage in actual use.—H. N. O.

Units of Electricity and Light. *Tech. News Bull., National Bureau of Standards.* The introduction of absolute units in electricity and the new system of units of light, which was to have taken place on January 1, 1940, in accordance with decisions of the International Committee on Weights and Measures, must now await resumption of normal international relations. Although the new methods of defining the units will be fundamentally different from the old, the actual size of the units will not be changed enough to affect seriously present commercial usage in this country. It would be possible to make the change independently, but one of the principal purposes of the proposed change is to obtain and to maintain closer agreement between the units used in different countries; this purpose would be defeated by separate action of one country.

Early in 1939 it became evident that final agreement upon values for the units could not be reached soon enough to permit their use at the beginning of 1940. In particular, the Physikalisch-Technische Reichsanstalt had not completed its absolute measurements of electric current, and some German authorities had questioned the validity of the spectral luminosity factors which had previously been accepted as the basis for photometry of lights differing in color from the primary standard (a blackbody at the temperature of freezing platinum). It was hoped, however, that these difficulties might be resolved to such an extent that the General Conference on Weights and Measures, meeting in October, could at least set a new date for adoption of the new units.

At meetings of the international Advisory Committee on Photometry and of the International Commission on Illumination, held in June, 1939, unanimous agreement was reached to retain the accepted luminosity factors and to complete the system of practical photometric standards derived from the platinum blackbody by means of those factors. International comparisons of gas-filled incandescent standard lamps were to be made, so that the new units could be introduced for all types of lamps in all countries on January 1, 1941.

On the electrical units no such definite recommendation could be made, because the German measurements of current were not finished and preliminary results were not concordant with those obtained in other countries. Arrangements were made for direct comparisons of apparatus between the Reichsanstalt and the National Bureau of Standards in order to find the cause of the discrepancies in results.

The war has of course prevented the execution of these plans. The meetings of the International Committee and the General Conference on Weights and Measures scheduled for October were canceled and no progress toward international agreement on the units can be made until peace is restored. In the meantime the Bureau is continuing experiments to confirm or correct its own determinations of the electrical units. In photometry, the results of international comparisons so far as they were completed were satisfactory.—D. R.

Check List of Periodical Literature

The scientific work of the second Byrd Antartic Expedition. T. C. Poulter; *Sci. Mo.* **49**, 5–20, July, 1939. An illustrated article by the physicist who was senior scientist and second in command of the expedition.

The exact sciences in a liberal education. G. P. Harnwell; *Sci. Mo.* **49**, 71–78, July, 1939.

Contemporary Advances in Physics, XXXII: Particles of the Cosmic Rays. K. K. Darrow; *Bell Sys. Tech. J.* **18**, 190–217, Jan., 1939.

Does science afford a basis for ethics? E. G. Conklin; *Sci. Mo.* **49**, 295–303, Oct., 1939. The only final solution of the problems that now threaten the very existence of civilization is through the cooperation of science, education and religion in the cultivation of a wider and more generous form of ethics.

New approaches to the science of voice. C. E. Seashore; *Sci. Mo.* **49**, 340–350, Oct., 1939. Brief descriptions of recent advances in a field in which more has been accomplished in the way of fundamental contributions toward rigorous scientific procedure in the past 20 years than in all previous history.

Rumford as a sociological engineer. C. H. Dwight; *Sci. Mo.* **49**, 504–508 (1939).

The outlook in fluid mechanics. W. F. Durand; *J. Frank. Inst.* **228**, 183–212 (1939).

Science and human affairs. W. F. G. Swann; *J. Frank. Inst.* **228**, 263–291 (1939).